Homework #12

Problem 1:

We know that

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx, \tag{1}$$

and

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x+a) e^{i\omega x} dx. \tag{1}$$

Antifourier transforming (1) we obtain:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega x} d\omega. \tag{3}$$

Then, according to (3)

$$f(x+a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega(x+a)} d\omega. \tag{4}$$

But

$$f(x+a) = g(x), \tag{5}$$

and

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{-i\omega x} d\omega. \tag{6}$$

Then, comparing the integrands in (4) and (6) which should be identical because of (5) we obtain:

$$G(\omega) = F(\omega) e^{-ia\omega}. \tag{7}$$