Homework #12

Problem 8:

a) We have to consider $f(x) = 1$ for $|x| < 1$ and 0 otherwise.
1) f(x) cannot be expanded in terms of the sine FT because it is an even function and the sine expansion can only represent odd functions.
2) I should use the cosine FT.

$$f_c(k) = \sqrt{\frac{2}{\pi}} \int_0^1 cos(kx)dx = \sqrt{\frac{2}{\pi}} \frac{sink}{k}. \tag{1}$$

b) Now consider $f(x) = 1$ for $a < x < b$ with $a > 0$ and $b > 0$ and $f(x) = 0$ otherwise.
1) I can expand f(x) in terms of the sine FT. I just have to assume that f(x) is odd, i.e., so that $f(x) = -1$ if $-b < x < -a$.
2) Now:

$$f_s(k) = \sqrt{\frac{2}{\pi}} \int_a^b sin(kx)dx = \sqrt{\frac{2}{\pi}} \frac{(coska - coskb)}{k}. \tag{2}$$

3) My sine FT is appropriated for $x \geq 0$.
4) As I mentioned in 1) I am assuming that $f(-x) = -1$ if $-b < -x < -a$.
5) Yes. I have to assume that $f(x)$ is even, i.e., that $f(-x) = 1$ if $-b < -x < -a$.
6) In this case I should use the exponential form of the FT. By doing this the sine and cosine expansion add each other for $x > 0$ but they cancel for $x < 0$. The coefficients of the expansion are given by:

$$f(k) = \sqrt{\frac{1}{2\pi}} \int_a^b e^{ikx} dx = \sqrt{\frac{1}{2\pi}} \frac{(e^{ikb} - e^{ika})}{k}. \tag{3}$$