Homework #2

Problem 1: Consider the two-dimensional vector $\mathbf{r}$ which has coordinates (x_1, x_2) in a cartesian system S so that $\mathbf{r} = x^i \hat{e}_i$. Remember that in S $x_i = x^i$.

a) Find expressions for x^i in terms of r and θ, i.e., the magnitude of the vector and the angle that it makes with the x_1-axis.

b) Now consider an oblique system S' with $\hat{e}'_1$ forming an angle γ with $\hat{e}_1$ and $\hat{e}'_2$ forming an angle β with $\hat{e}_2$. Show that $\hat{e}'_j = A^j{}_i \hat{e}_j$ and provide the explicit form of the matrix $A^j{}_i$ in terms of the angles β and γ.

c) Find the covariant components x'_i of $\mathbf{r}$ in S' and show that they transform as $x'_i = A^j{}_i x_j$.

d) Find the contravariant components x'^i of $\mathbf{r}$ in S' and show that they transform as $x'^i = M^i{}_j x^j$. Provide the explicit form for $M^i{}_j$ and verify that $A^i{}_k M^k{}_j = \delta^i{}_j$.

e) Show that $A^i{}_j = \frac{\partial x^i}{\partial x'^j}$ and $M^i{}_j = \frac{\partial x'^i}{\partial x^j}$.

f) Find the values of β and γ for which S' reduces to the oblique system used in class with $\hat{e}'_1$ parallel to $\hat{e}_1$ and making an angle α with $\hat{e}'_2$.