Problem 3:

We know that $\mathbf{e}_i = \mathbf{e}_i(q_1, q_2, q_3)$ and $|\mathbf{e}_i| = 1$. We also know from the hint that $\frac{\partial \mathbf{e}_i^2}{\partial q_j} = 0$ then

$$0 = \frac{\partial \mathbf{e}_i^2}{\partial q_j} = \frac{\partial (\mathbf{e}_i \cdot \mathbf{e}_i)}{\partial q_j} = 2 \frac{\partial \mathbf{e}_i}{\partial q_j} \cdot \mathbf{e}_i.$$

The above occurs only if $\frac{\partial \mathbf{e}_i}{\partial q_j} = 0$ or if $\frac{\partial \mathbf{e}_i}{\partial q_j}$ is orthogonal to \mathbf{e}_i .