Homework #3

Problem 5:

We know that

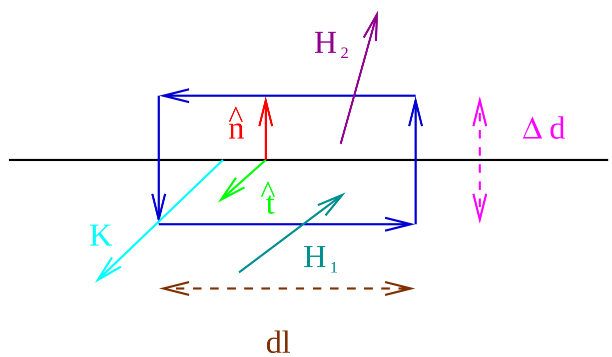
$$\nabla \times \mathbf{H} = \mathbf{J} \tag{1}$$

and

$$\mathbf{K} = lim_{\Delta d \to 0, J \to \infty} \mathbf{J} \Delta d. \tag{2}$$

Notice that as Δd goes to 0, $\mathbf{J}$ which is the total current per unit area will diverge, but the product of $\mathbf{J}$ and Δd will remain constant.

Let's consider the loop shown in the figure, which is in a plane perpendicular to the linear density of current $\mathbf{K}$, and apply Stokes' theorem:



$$\int_S \nabla \times \mathbf{H}.d\mathbf{S} = \oint_C \mathbf{H}.d\mathbf{l}. \tag{3}$$

If we let $\Delta d \to 0$ and use (1), (3) becomes:

$$\mathbf{J}\Delta d.\hat{\mathbf{t}}dl = (\mathbf{H}_2 - \mathbf{H}_1).d\mathbf{l}. \tag{4}$$

In the figure notice that

$$d\mathbf{l} = dl\hat{\mathbf{t}} \times \hat{\mathbf{n}}. \tag{5}$$

Then replacing (2) and (5) in (4) we obtain:

$$\mathbf{K}.\hat{\mathbf{t}} = (\mathbf{H}_2 - \mathbf{H}_1).\hat{\mathbf{t}} \times \hat{\mathbf{n}}. \tag{6}$$

Since $\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = \mathbf{b}.(\mathbf{c} \times \mathbf{a})$ (6) becomes

$$\mathbf{K}.\hat{\mathbf{t}} = \hat{\mathbf{t}}.[\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1)]. \tag{7}$$

Now notice that $\mathbf{K}$ is parallel to $\hat{\mathbf{t}}$ by construction. However, the components of the magnetic field in the plane perpendicular to the normal $\hat{\mathbf{n}}$ do not need to be parallel nor orthogonal to $\mathbf{K}$. Considering a loop in the plane of $\mathbf{K}$ we notice that the tangential components of the magnetic field parallel to $\mathbf{K}$ have to be continuous which means that only the tangential components of $\mathbf{H}$ normal to $\mathbf{K}$ are discontinuous. Then Eq.(7) can be expressed as:

$$\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}, \tag{8}$$

which indicates that the tangential components of $\mathbf{H}$ perpendicular to $\mathbf{K}$ are discontinuous.