Homework #2

Problem 6 - 3.9.1:

We know that

$$\mathbf{F} = r^{2n}(x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}) = r^{2n}\mathbf{r} = r^{2n}r\hat{\mathbf{r}} = r^{2n+1}\hat{\mathbf{r}} = f(r)\hat{\mathbf{r}}. \tag{1}$$

a)

$$\nabla.\mathbf{F} = \sum_{i=1}^{3} \frac{\partial F_i}{\partial x_i} = (3 + 2n)r^{2n}. \tag{2}$$

b) We saw in class that if $\mathbf{F} = \mathbf{r}f(r)$ then $\nabla \times \mathbf{F} = 0$, since this is the case for our function with $f(r) = r^{2n+1}$ we have that

$$\nabla \times \mathbf{F} = 0. \tag{3}$$

c) From Eq.(1) we see that in order to find the potential we can calculate

$$\phi = \int f(r)dr = \int r^{2n+1}dr. \tag{4}$$

If $n \neq -1$ then

$$\phi = \frac{-r^{2n+2}}{2(n+1)}, \tag{5}$$

if $n = -1$

$$\phi = -lnr. \tag{6}$$

d)ϕ diverges at 0 and ∞ if $n = -1$.