Problem 6-3.9.1:

We know that

$$
\begin{equation*}
\mathbf{F}=r^{2 n}(x \hat{\mathbf{x}}+y \hat{\mathbf{y}}+z \hat{\mathbf{z}})=r^{2 n} \mathbf{r}=r^{2 n} r \hat{\mathbf{r}}=r^{2 n+1} \hat{\mathbf{r}}=f(r) \hat{\mathbf{r}} \tag{1}
\end{equation*}
$$

a)

$$
\begin{equation*}
\nabla \cdot \mathbf{F}=\sum_{i=1}^{3} \frac{\partial F_{i}}{\partial x_{i}}=(3+2 n) r^{2 n} \tag{2}
\end{equation*}
$$

b) We saw in class that if $\mathbf{F}=\mathbf{r} f(r)$ then $\nabla \times \mathbf{F}=0$, since this is the case for our function with $f(r)=r^{2 n+1}$ we have that

$$
\begin{equation*}
\nabla \times \mathbf{F}=0 \tag{3}
\end{equation*}
$$

c) From Eq.(1) we see that in order to find the potential we can calculate

$$
\begin{equation*}
\phi=\int f(r) d r=\int r^{2 n+1} d r \tag{4}
\end{equation*}
$$

If $n \neq-1$ then

$$
\begin{equation*}
\phi=\frac{-r^{2 n+2}}{2(n+1)} \tag{5}
\end{equation*}
$$

if $n=-1$

$$
\begin{equation*}
\phi=-\ln r . \tag{6}
\end{equation*}
$$

d) $\phi$ diverges at 0 and $\infty$ if $n=-1$.

