Homework #3

Problem 7:

i) Let's find the contravariant basis vectors in K'. We know that the covariant basis vectors of K' are given by:

$$\hat{\mathbf{e}'}_1 = (1, 0) = \hat{\mathbf{e}}_1, \tag{1}$$

and

$$\hat{\mathbf{e}'}_2 = (\cos\alpha, \sin\alpha) = \cos\alpha\hat{\mathbf{e}}_1 + \sin\alpha\hat{\mathbf{e}}_2. \tag{2}$$

Now let's find $\hat{\mathbf{e}'}^2$ knowing that

$$\hat{\mathbf{e}'}^2.\hat{\mathbf{e}'}_1 = (a, b).(1, 0) = a = 0. \tag{3}$$

We see that $\hat{\mathbf{e}'}^2 = (0, b)$ and we find b by requesting that

$$\hat{\mathbf{e}'}^2.\hat{\mathbf{e}'}_2 = (0, b).(\cos\alpha, \sin\alpha) = b\sin\alpha = 1. \tag{4}$$

Then, $b = \text{cosec}\alpha$ and then

$$\hat{\mathbf{e}'}^2 = (0, \text{cosec}\alpha). \tag{5}$$

Notice that $\hat{\mathbf{e}'}^2$ is not normalized to 1.

Now let's find $\hat{\mathbf{e}'}^1$ knowing that

$$\hat{\mathbf{e}'}^1.\hat{\mathbf{e}'}_1 = (c, d).(1, 0) = c = 1. \tag{6}$$

We see that $\hat{\mathbf{e}'}^1 = (1, d)$ and we find d by requesting that

$$\hat{\mathbf{e}'}^1.\hat{\mathbf{e}'}_2 = (1, d).(\cos\alpha, \sin\alpha) = (\cos\alpha + d\sin\alpha) = 0. \tag{7}$$

Then, $d = -\text{cotan}\alpha$ and then

$$\hat{\mathbf{e}'}^1 = (1, -\text{cotan}\alpha). \tag{8}$$

ii) Now we need to find the components of $\mathbf{r}'$ in the contravariant (or dual) basis. We know that

$$\mathbf{r} = x_1\hat{\mathbf{e}}_1 + x_2\hat{\mathbf{e}}_2 = s\hat{\mathbf{e}'}^1 + t\hat{\mathbf{e}'}^2. \tag{9}$$

Replacing Eq. 5 and 8 in Eq. 9 I can compare components so that I find the values of s and t in terms of x_1 and x_2. I obtain:

$$x_1 = s, \tag{10}$$

and

$$x_2 = -s\text{cotan}\alpha + t\text{cosec}\alpha. \tag{11}$$

Then, from 10 and 11 we find that

$$s = x_1, \tag{12}$$

and

$$t = x_1 \cos\alpha + x_2 \sin\alpha. \tag{13}$$

Comparing with the expression that we got in class for x'_i we see that $s = x'_1$ and $t = x'_2$ and thus

$$\mathbf{r}' = x'_1 \hat{\mathbf{e}'}^1 + x'_2 \hat{\mathbf{e}'}^2. \tag{14}$$