Homework #4

Problem 4:

Consider the vectors $V^i = (V^1, V^2)$ and $W^i = (W^1, W^2)$ defined in the system S given by the orthogonal coordinates (x^1, x^2). Consider the transformation to system S' given by the oblique axis (x'^1, x'^2). x'^1 is parallel to x^1 and x'^2 makes an angle α with x'^1. Thus, as in the example done in class, the transformation matrices are given by:

$$U^i{}_j = \frac{\partial x'^i}{\partial x^j} = \begin{pmatrix} 1 & -\mathrm{cotan}\alpha \\ 0 & \frac{1}{\sin\alpha} \end{pmatrix}, \tag{1}$$

and

$$(U^{-1})^i{}_j = \frac{\partial x^i}{\partial x'^j} = \begin{pmatrix} 1 & \cos\alpha \\ 0 & \sin\alpha \end{pmatrix}, \tag{2}$$

a) Construct

$$A^{ij} = V^i W^j$$

$$A_{ij} = V_i W_j$$

$$A^i{}_j = V^i W_j$$

$$A_i{}^j = V_i W^j$$

b) Find in S' A'^{ij}, $A'^i{}_j$, A'_{ij}, and $A'^{\,j}_i$.

c) We know that in S, $A^{ij} = A^i{}_j = A_{ij} = A_i{}^j$ because the system is cartesian. Using the expression for the matrices U, U^{-1} and A calculate A' by performing the similarity transformation $A' = UAU^{-1}$.

Now compare the expression obtained with the expressions for A' obtained in part b). Which one corresponds to the similarity transformation?

d) Based on the results obtained in c) write the similarity transformation using tensor notation.