Homework #5

Problem 1 - 4.1.8:

 $T_{ijk...}$ is a tensor of rank n.

$$\sum_{j} \frac{\partial T_{ijk\dots}}{\partial x_j} = \partial^j T_{ijk\dots} = C_{i\hat{j}k\dots},$$

where \hat{j} indicates that the index j is no longer there. Since the derivative is a tensor of rank 1 its direct product with T gives a tensor of rank n+1, the contraction of the index j reduces the rank of this tensor by 2, i.e. n+1-2=n-1. Thus C is a tensor of rank n-1. Notice that since we are using cartesian coordinates we have not used covariant and contravariant placements for the indices.