Homework #9

Problem 1 - 9.7.1:

We have to solve:

$$\frac{\partial T(r,t)}{\partial t} = K\nabla^2 T. \tag{1}$$

Since the b.c. are given on a sphere we will use spherical coordinates and using separation of variables we will propose a solution of the form

$$T(r,t) = R(r)T(t), \tag{2}$$

since there is no angular dependence. Now writing (1) in spherical coordinates, plugging (2) in (1) and dividing by (2) we obtain:

$$\frac{1}{TK}\frac{\partial T}{\partial t} = \frac{1}{Rr^2}\frac{\partial}{\partial r}(r^2\frac{\partial R}{\partial r}) = -\alpha^2, \tag{3}$$

where we have equated each term which depends only on t or r to the separation constant $-\alpha^2$. T satisfies the harmonic oscillator equation so

$$T \propto e^{i\sqrt{K}\alpha t}, \tag{4}$$

and rearranging the first term of (3) we see that R satisfies:

$$r^2\frac{\partial^2 R}{\partial r^2} + 2r\frac{\partial R}{\partial r} + \alpha^2 r^2 R = 0. \tag{5}$$

We see that (5) is the Bessel equation with $n = 0$.