Homework #9

Problem 5 - 15.2.3:

We want to expand the delta function in Legendre polynomials which is OK because they form a set of orthogonal functions in the interval [-1,1]. So we write:

$$\delta(x) = \sum_{l=0}^{\infty} A_l P_l(x). \tag{1}$$

To find the coefficients A_l we use the orthogonality properties of the P_l's. We multiply both sides of (1) by $P_n(x)$ and we integrate between -1 and 1:

$$\int_{-1}^{1} \delta(x) P_n(x) dx = \sum_{l=0}^{\infty} A_l \int_{-1}^{1} P_n(x) P_l(x) dx, \tag{2}$$

which becomes

$$P_n(0) = A_n \frac{2}{2n+1}. \tag{3}$$

Then

$$A_n = \frac{2n+1}{2} P_n(0). \tag{4}$$

An expression for $P_n(0)$ is given in the Hint. We see that only terms with even n survive. Replacing in (1) we obtain:

$$\delta(x) = \sum_{j=0}^{\infty} (2j + \frac{1}{2}) \frac{(-1)^j (2j)!}{2^{2j} (j!)^2} P_{2j}(x). \tag{5}$$