Homework #9

Problem 7 - 15.2.13:

We know that

$$\mathbf{E} = -\nabla\Phi. \tag{1}$$

In spherical coordinates

$$E_r = \frac{\partial\Phi}{\partial r}, \tag{2}$$

$$E_\theta = \frac{1}{r}\frac{\partial\Phi}{\partial\theta}, \tag{3}$$

and

$$E_\phi = \frac{1}{r sin\theta}\frac{\partial\Phi}{\partial\phi} = 0, \tag{4}$$

since Φ is independent of ϕ.

In the previous problem we found that

$$\Phi(r,\theta) = \frac{q}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^{2n}(n!)^2}\frac{r_<^{2n}}{r_>^{2n+1}}P_{2n}(cos\theta). \tag{5}$$

The derivatives with respect to r are straightforward. The derivative with respect to θ only affects the $P_l(cos\theta)$ but from (15.79) we find that $P_l'(cos\theta) = -P_l^1(cos\theta)$. Then we obtain for $r < a$:

$$E_r = \frac{q}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^{2n}a^{2n+1}(n!)^2}r^{2n-1}P_{2n}(cos\theta), \tag{6}$$

and

$$E_\theta = -\frac{q}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^{2n}a^{2n+1}(n!)^2}r^{2n-1}P_{2n}^1(cos\theta). \tag{7}$$

and for $r > a$:

$$E_r = -\frac{q}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^{2n}r^{2n+2}(n!)^2}a^{2n}P_{2n}(cos\theta). \tag{8}$$

and

$$E_\theta = -\frac{q}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^{2n}r^{2n+2}(n!)^2}a^{2n}P_{2n}^1(cos\theta). \tag{9}$$