Midterm Exam

P571

October 2, 2014

SHOW ALL WORK TO GET FULL CREDIT!

WARNING!!! Points will be taken if numerical calculations are not provided and if calculations are left just indicated.

PART I: **DO IT IN CLASS** Turn your work in before leaving. Take the printed copy of the test home.

PART II: Take the test home and bring **ALL** the questions solved on Tuesday October 7. Your grade for the test will be the **sum of the two** parts. The point value of each question is indicated in the test. A perfect score is worth 200 points as a result of 50 points to be earned in class and 150 points to be earned at home. If you are 100% sure about the work you did in class, you do not need to redo it at home. In that case the points obtained in class will be counted twice.

PART I

Consider a cartesian system S in 2-dimensional space where the prototype contravariant vector is given by $\mathbf{r} = x^i \mathbf{e}_i$ with $(x^1, x^2) = (x, y)$ and a coordinate system S', also two-dimensional, with coordinates $(x'^1, x'^2) = (2x^1 - x^2, -x^1 + 2x^2)$ so that in S' the prototype contravariant vector is given by $\mathbf{r}' = x'^i \mathbf{e}'_i$.

a) Write the transformation matrix $M^i{}_j = \frac{\partial x'^i}{\partial x^j}$.(8 points.)

b) Write the transformation matrix $A^i{}_j = \frac{\partial x^i}{\partial x'^j}$ and verify that $A^i{}_j$ is the inverse of $M^i{}_j$.(10 points.)

c) Write expressions for the vectors $\mathbf{e}_i$ that form the covariant basis in S.(2 points.) .

d) Write expressions for the vectors $\mathbf{e}'_i$ that form the covariant basis in S'. Provide expressions for them in the orthogonal system. (10 points.) Hint: remember that as a vector $\mathbf{r} = \mathbf{r}'$.

d-i) Are these basis vectors orthogonal? Why? (2 points.)

d-ii) Are these basis vectors normal? Why? (2 points.)

e) Make a figure showing the basis vectors $\mathbf{e}_i$ and $\mathbf{e}'_i$.(6 points.)

f) Calculate the metric tensor g'_{ij} in S'.(10 points.)

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PART II

g) Write expressions for the vectors $\mathbf{e}'^i$ that form the contravariant (or dual) basis in S'. Provide expressions for them in the orthogonal system.(10 points.)

g-i) Draw a figure showing the vectors.(5 points.)

h) Now calculate the covariant components x'_i of $\mathbf{r}'$. Express you result in terms of the components x^i of $\mathbf{r}$ in the cartesian system S.(10 points.)

Now consider the vectors $\mathbf{p}'$ and $\mathbf{k}'$ given in S' by

$$\mathbf{p}' = 3\mathbf{e}'_1 - 1\mathbf{e}'_2,$$

and

$$\mathbf{k}' = -2\mathbf{e}'_1 + 1\mathbf{e}'_2.$$

i) Calculate the covariant components of $\mathbf{p}$ and $\mathbf{k}$ in system S'.(10 points.)

j) Obtain, in system S', the magnitude of $\mathbf{p}$ and $\mathbf{k}$ and find the angle between the two vectors.(10 points.)
j-i)Will your answer be different in S? Why? Provide expressions for p^i and k^i in S to check your answer. (10 points.)

Now consider the tensor $T'^{ijlm} = k'^i k'^j p'^l p'^m$ in S'.

k) What is the rank of the tensor T'^{ijlm}? (5 points.)
k-i) How many components does the tensor have? (10 points)
k-ii) How many of these components are independent? Why? (10 points)

l) What is the rank of $T'^{i\ j}_{i\ j}$? Why? (10 points.)

l-i) Provide the explicit numerical value of the tensor $T'^{i\ j}_{i\ j}$ and compare it with $T^{i\ j}_{i\ j}$. Did you expect this result? Why? (10 points.)