Midterm Exam II

P571

November 13, 2014

SHOW ALL WORK TO GET FULL CREDIT!
WARNING!!! Points will be taken if numerical calculations are not performed and if calcula tions are left just indicated.

PART I: **ONLY ONE OF THE THREE PROBLEMS WILL BE GRADED**. Take a look at the 3 problems. Each of them is worth 25 points. To make sure that you have enough time to do your work you will have to turn in only **ONE** of the 3 problems. **If you turn more than 1 problem only the one on top will be graded and 5 points will be deducted from your grade.**

PART II: Take the test home and bring **ALL** the problems solved on **Tuesday November 18**. Your grade for the test will be the **sum of the two** parts. A perfect score is worth 100 points.

Problem 1: Consider the vector

$$\mathbf{A}(\mathbf{x}) = \frac{\mathbf{m} \times \mathbf{x}}{|\mathbf{x}|^3}, \tag{1}$$

where $\mathbf{x}$ is the contravariant vector position x^i, $\mathbf{m}$ is a pseudovector with contravariant components m^j independent of $\mathbf{x}$, and $|\mathbf{x}|$ is the magnitude of $\mathbf{x}$ given by $|\mathbf{x}| = (x_i x^i)^{1/2}$.

a) Write $\mathbf{A}$ using tensor notation, with correctly placed covariant and contravariant indices.(5 points)

b) Did you obtain the contravariant or covariant components of $\mathbf{A}$ in part (a)? Why? (2.5 points)

c) Is $\mathbf{A}$ a tensor or a pseudotensor? Why? (2.5 points)

d) Write in tensor notation an expression for the contravariant components of the curl of $\mathbf{A}$, i.e., provide a tensor expression for $(\nabla \times \mathbf{A})^i$ using the form of $\mathbf{A}$ given in Eq.(1). (2.5 points)

e) Is the curl of $\mathbf{A}$ a tensor or a pseudotensor? Why? (2.5 points)

f) Using tensor notation show that

$$\nabla \times \mathbf{A} = \frac{3\mathbf{n}(\mathbf{n}.\mathbf{m}) - \mathbf{m}}{|\mathbf{x}|^3}, \tag{2}$$

where $\mathbf{n} = \frac{\mathbf{x}}{|\mathbf{x}|}$. Clearly show ALL your steps. You may need to use the metric tensor g_{ij} to change from covariant to contravariant components or vice versa. (10 points)

Problem 2: Consider the equation

$$\epsilon^{\alpha\beta\gamma\rho}\partial_\beta F_{\gamma\rho} = 0, \tag{3}$$

where $F_{\gamma\rho}$ is the covariant field-strength tensor in Minkowsky space given by

$$F_{\gamma\rho} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}, \tag{4}$$

and E_i and B_i are the 3 components of the electric and magnetic field and $\epsilon^{\alpha\beta\gamma\rho}$ is the Levi-Civita tensor.

a) What is the rank of the tensor represented by the left hand side of Eq.(3)? Why? (5 points).

b) Find the explicit form of Eq(3) for $\alpha = 0$. (5 points)

c) A general Lorentz transformation from a system S at rest to a system S' moving with velocity $\mathbf{v}$ with respect to S is given by

$$x'^0 = \gamma(x^0 - \vec{\beta}.\mathbf{x}), \tag{5}$$

$$\mathbf{x}' = \mathbf{x} + \frac{(\gamma - 1)}{\beta^2}(\vec{\beta}.\mathbf{x})\vec{\beta} - \gamma\vec{\beta}x^0, \tag{6}$$

where $\vec{\beta} = \mathbf{v}/c$ and $\gamma = (1 - \beta^2)^{-1/2}$. If $\mathbf{v} = (\frac{2}{3}c, 0, \frac{1}{3}c)$ provide the transformation matrix $M^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu}$. Note: all the elements of the matrix have to be numbers of the form a or b/c, in other words, do the calculations and simplify your expressions as much as possible. (5 points).

d) Find F'^{01} in system S' if in system S the electric field is $\mathbf{E} = (0, 0, E)$ and the magnetic field is $\mathbf{B} = (0, 0, 0)$. (5 points). Hint: it may be helpful to write $F^{\alpha\beta}$ in terms of $F_{\gamma\epsilon}$.

e) Find B'_y the y-component of the magnetic field $\mathbf{B}'$ in S'. (5 points).

Problem 3: Consider a spherical shell or radius a centered at the origin with a surface potential given by $\Phi_s = V_0 \sin^2(\theta)$.

a) Find the electric potential $\Phi(\mathbf{r})$ inside and outside the shell. (10 points)

b) Now consider that the surface of part (a) is surrounded by a grounded sphere of radius b ($b > a$) also centered at the origin.

i) Find the electric potential in all space. (10 points)

ii) Has the potential inside the sphere of radius a changed after the grounded sphere was added? (2.5 points)

iii) For what value of b the result for part (b) of this problem will be the same as the result for part (a)? Use this to check your answer to part (i). (2.5 points)