Midterm Exam

P551

October 8, 2015

SHOW ALL WORK TO GET FULL CREDIT!

WARNING!!! Points will be taken if numerical calculations are nor provided and if calculations are left just indicated.

PART I: **DO IT IN CLASS** Turn your work in before leaving. Take the printed copy of the test home.

PART II: Take the test home and bring **ALL** the questions solved on Tuesday October 13. Your grade for the test will be the **sum of the two** parts. Each question is worth 5 points. A perfect score is worth 200 points as a result of 55 points to be earned in class and 145 points to be earned at home. If you are 100% sure about the work you did in class, you do not need to redo it at home. In that case the points obtained in class will be counted twice.

PART I

Problem 1: Consider N molecules that are stuck on a metal surface of square symmetry. Each molecule can either lie flat on the surface of the metal, in which case it must be aligned to one of two directions, x and y, or it can stand up along the z direction. There is an energy cost of $\epsilon > 0$ associated with the molecule standing up, and zero energy for molecules lying flat along x or y directions.

a) For the case of one single molecule, i.e., $N = 1$,

i) How many microstates does the system have?

ii) Provide the energy of each microstate.

b) Now consider the case of N molecules,

i) How many microstates does the system have?

ii) What is the smallest possible value that the energy of this system can have?

iii) How many microstates are accessible to the system when it is in the macrostate with the lowest energy, i.e., how many microstates have the lowest energy?

iv) What is the largest value of the energy that this system can have?

v) How many microstates are accessible to the system when it is in the macrostate with the highest energy, i.e., how many microstates have the highest energy?

c) This part will be done at home.

d) Now consider that the metal with the N molecules on its surface is placed at room temperature (T=300K).

i) What statistical ensemble would be the most appropriate to study the properties of the system in these conditions? Why?

ii) Write the partition function Z for the system of N particles.

Problem 2: Consider a classical ideal gas made of N monoatomic molecules which are confined to move in 2 dimensions.

a) Write an expression for the energy of one molecule in the gas in terms of its coordinates $\{p_i, q_i\}$.

b) Using the principle of equipartition of the energy obtain the energy of the gas per molecule in terms of its temperature T.

STOP HERE!!!!: Hand your work before leaving and take home the printed copy of the test. Bring **ALL** the questions answered on Tuesday October 8.

PART II

Problem 1: Consider N molecules that are stuck on a metal surface of square symmetry. Each molecule can either lie flat on the surface of the metal, in which case it must be aligned to one of two directions, x and y, or it can stand up along the z direction. There is an energy cost of $\epsilon > 0$ associated with the molecule standing up, and zero energy for molecules lying flat along x or y directions.

a) For the case of one single molecule, i.e., $N = 1$,

i) How many microstates does the system have?

ii) Provide the energy of each microstate.

b) Now consider the case of N molecules,

i) How many microstates does the system have?

ii) What is the smallest possible value that the energy of this system can have?

iii) How many microstates are accessible to the system when it is in the macrostate with the lowest energy, i.e., how many microstates have the lowest energy?

iv) What is the largest value of the energy that this system can have?

v) How many microstates are accessible to the system when it is in the macrostate with the highest energy, i.e., how many microstates have the highest energy?

c) Now consider that the system of N molecules is prepared so that it is in a macrostate with energy $E = n_z\epsilon$ where n_z indicates the number of molecules that are standing up along the z axis.

i) How many microstates are accessible to the system in this case?

ii) Using the expression that you obtained in the previous question (c-i) say how many microstates are accessible to the system when 1) $n_z = 0$ and when 2) $n_z = N$? Compare with b-iii and b-v.

d) Now consider that the metal with the N molecules on its surface is placed at room temperature (T=300K).

i) What statistical ensemble would be the most appropriate to study the properties of the system in these conditions? Why?

ii) Write the partition function Z for the system of N particles.

iii) Obtain the average energy $< E >$ of the system at room temperature. Use that $\epsilon = 0.01$ eV, $k = 8.617 \times 10^{-5}$eV/K, and $N = N_A = 6.022 \times 10^{23}$.

iv) Find the average energy $< E >$ of the system when $T \to 0$. Use that $\epsilon = 0.01$ eV, $k = 8.617 \times 10^{-5}$eV/K, and $N = N_A = 6.022 \times 10^{23}$.

v) Find the average energy $< E >$ of the system when $T \to \infty$. Use that $\epsilon = 0.01$ eV, $k = 8.617 \times 10^{-5}$eV/K, and $N = N_A = 6.022 \times 10^{23}$.

vi) Does the result you obtained in (v) correspond to the maximum energy that the system can have? Explain why this result is sensible.

e) Obtain an expression for the average number $< n_z >$ of molecules standing up as a function of β and find out

i) What fraction of the molecules $< n_z > /N$ are standing up at room temperature?

ii) What fraction of the molecules $< n_z > /N$ are standing up when $T \to 0$?

iii) What fraction of the molecules $< n_z > /N$ are standing up when $T \to \infty$?

iv) Is it possible to make all the molecules stand up by cranking up the temperature? Why?

f) If you were told that the metal at room temperature has N_A particles on average, i.e., $< N >= N_A$,

i) What statistical ensemble would be the most appropriate to study the properties of the system in these conditions? Why?

ii) Write the grand-partition function $\mathcal{Z}$ of the system.

iii) Find the chemical potential μ of the system when it is at room temperature and has $< N >= N_A$.

Problem 2: Consider a classical ideal gas made of N monoatomic molecules which are confined to move in 2 dimensions.

a) Write an expression for the energy of one molecule in the gas in terms of its coordinates $\{p_i, q_i\}$.

b) Using the principle of equipartition of the energy obtain the average energy of the gas per molecule in terms of its temperature T.

c) Based on the result obtained in part (b) what should be $< v^2 >$?

d) Now, as we did in class for the gas in 3D, find the speed distribution function $F(v)$ for the 2D gas.

e) Using $F(v)$ obtain

i) $< v >$,

ii) $< v^2 >$ (compare with (c)) and v_{rms},

iii) $\tilde{v}$, i.e. the most probable speed.