Second Midterm Exam

P551

November 12, 2015

SHOW ALL WORK TO GET FULL CREDIT!

WARNING!!! Points will be taken if numerical calculations are nor provided and if calculations are left just indicated.

PART I: **DO IT IN CLASS** Turn your work in before leaving. Take the printed copy of the test home.

PART II: Take the test home and bring **ALL** the questions solved on Tuesday November 17. Your grade for the test will be the **sum of the two** parts. Each question is worth 5 points. A perfect score is worth 110 points as a result of 35 points to be earned in class and 75 points to be earned at home. If you are 100% sure about the work you did in class, you do not need to redo it at home. In that case the points obtained in class will be counted twice.

## PART I

**Problem 1**: Consider a system of 4 non-interacting particles without spin. Assume that the particles are inside a container where the allowed single particle energy levels are non-degenerate and evenly spaced with $\Delta e = \epsilon$ and $e_0 = 0$ for the lowest state.

a) Draw the first 5 single particle energy levels indicating the energy of each level in terms of $\epsilon$.

b) Provide the energy $E_0$ (in terms of $\epsilon$) and the degeneracy of the ground state of the system if the 4 particles are:

i) Identical (spinless) fermions. In this case provide also the Fermi energy $\mu_0$ in terms of $\epsilon$.

ii) Identical bosons.

iii) Distinguishable particles.

c)Suppose now that the system of 4 particles is prepared with energy $E = 2\epsilon$. Describe the number of accessible states for the system with their degeneracy if the particles are:

i) Identical (spinless) fermions.

ii) Identical bosons.

iii) Distinguishable particles.

**STOP HERE!!!!**: Hand your work before leaving and take home the printed copy of the test. Bring **ALL** the questions answered on Tuesday October 17.

## PART II

**Problem 1**: Consider a system of 4 non-interacting particles without spin. Assume that the particles are inside a container where the allowed single particle energy levels are non-degenerate and evenly spaced with $\Delta e = \epsilon$ and $e_0 = 0$ for the lowest state.

a) Draw the first 5 single particle energy levels indicating the energy of each level in terms of $\epsilon$.

b) Provide the energy $E_0$ (in terms of $\epsilon$) and the degeneracy of the ground state of the system if the 4 particles are:

i) Identical (spinless) fermions. In this case provide also the Fermi energy in terms of $\epsilon$.

ii) Identical bosons.

iii) Distinguishable particles.

c)Suppose now that the system of 4 particles is prepared with energy $E = 2\epsilon$. Describe the number of accessible states for the system with their degeneracy if the particles are:

i) Identical (spinless) fermions.

ii) Identical bosons.

iii) Distinguishable particles.

d) Now assume that the system is held at temperature $T = \epsilon/(10k)$ and $< N >= 4$. Calculate the average population $< n_s >$ of the level with energy $e_0 = 0$ and the level with $e_4 = 4\epsilon$ if the particles are:

i) Identical (spinless) fermions.You can assume that $\mu = \mu_0$.

ii) Identical bosons. Hint: Find the value of $\mu$ in terms of $\epsilon$ assuming that $|\mu| \ll kT$.

**Problem 2**: Consider a 2 dimensional solid formed by $N$ atoms arranged at the sites of a square lattice, i.e. in equilibrium each atom is separated by a distance $a_0$ from its 4 neighbors. Consider that the atoms can only vibrate about their equilibrium position along the $x$ and the $y$ direction (no vibrations along $z$ are allowed).

a) Write the Hamiltonian that describes the vibrational motion of the atoms in the 2D crystal.

b) Expand the potential energy term up to second order about the equilibrium position of the atoms and show that the Hamiltonian can be written in terms of one dimensional harmonic oscillator. Clearly indicate the number of independent oscillators.

c) Write the partition function for the system of harmonic oscillators.

d) Find an expression for $\sigma(\omega)d\omega$, the density of normal modes with frequency $\omega$ in the interval $(\omega, \omega + d\omega)$ under Debye's approximation for this 2-dimensional system. Clearly say how $\sigma(\omega)$ depends on $\omega$ and compare with the 3D case.

e) Calculate the heat capacity at constant volume for this system providing a explicit value for

i) $kT \ll \hbar\omega$ and compare with the 3D result.

ii) $kT \gg \hbar\omega$ and compare with the 3D result.