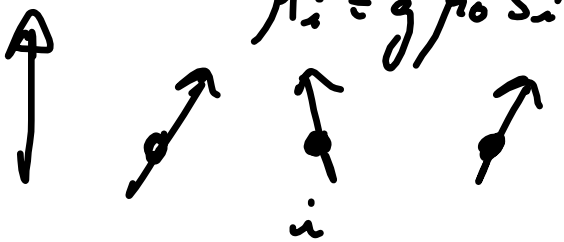
11/19

Last time:

Ferromagnetism

$H_0 \parallel \hat{z}$

$\bar{\mu}_i = g\mu_0 \bar{S}_i$



N spins S.

nearest neighbors.

$$\mathcal{H} = -g\mu_0 \sum_{j=1}^{N} \bar{S}_j \cdot \bar{H}_0 + \frac{1}{2}\left(-2J \sum_{j=1}^{N} \sum_{k=1}^{n} S_j^z S_k^z\right)$$

For a single spin:

$$\mathcal{H}_j = -g\mu_0 S_j^z H_0 - 2J \underbrace{\sum_{k=1}^{n} S_k^z S_j^z}_{\text{mean part.}}$$

Define

$$① \quad z J \langle \sum_{k=1}^{M} S_k^z \rangle \equiv g \mu_0 H_m$$

mean-field.

Then

$$\mathcal{H}_j = - g \mu_0 S_j^z (H_0 + H_m)$$

single particle problem that we solved.

Solution:

$$E_m = - g \mu_0 (H_0 + H_m) m_s$$

$m_s : -S, -S+1, \ldots S$

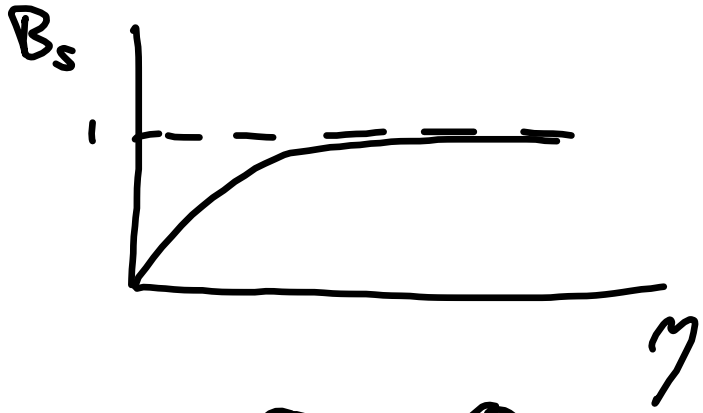
$2S+1$ values,

and

$$② \quad \langle S_j^z \rangle = S B_S(\eta)$$

Brillouin function.

$$\eta = \beta g \mu_0 (H_0 + H_m) \qquad \Rightarrow H_m = \frac{\eta}{g\mu_0\beta} - H_0 \quad (3)$$



Using (2) in (1):

$$2J \left\langle \sum_{k=1}^{n} S_k^z \right\rangle = 2J \sum_{k=1}^{n} \langle S_k^z \rangle =$$

$$= 2Jn S B_s(\eta) = g\mu_0 H_m \quad (4)$$

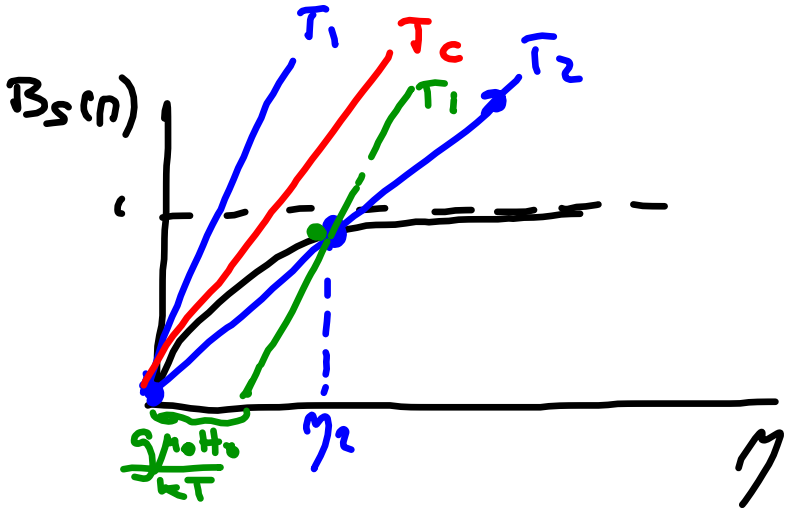
Rearranging (4)

using (3)

$$B_s(\eta) = \frac{g\mu_0 H m}{2JSm} = \cancel{g\mu_0}\,\frac{1}{\beta \cancel{g\mu_0}}\left(\frac{\eta - \beta g\mu_0 H_0}{2JSm}\right)$$

(5) $$= \frac{kT}{2JSm}\left(\eta - \frac{g\mu_0 H_0}{kT}\right)$$

if $H_0 > 0$ $\eta \neq 0$.



$H_0 = 0 \Rightarrow B_s(\eta) = \frac{kT}{2JSm}\eta$

For T_1 the only solution is $\eta = 0$

For T_2 there is a finite solution and so the system is ferromagnetic even if $H_0 = 0$.

The highest T at which $\eta > 0$ is found is called the Curie temperature T_c and it is obtained by requiring that the slope of $y = \frac{kT}{2JSn}\eta$ equals $\left.\frac{dB_s}{d\eta}\right|_{\eta=0}$.

$$\langle M \rangle = g\mu_0 \sum_{j=1}^{N} \langle S_j^z \rangle = g\mu_0 N S B_s(\eta) \quad (*)$$

To find the non trivial solution for $H_0 = 0$:

$$\left.\frac{dB_s}{d\eta}\right|_{\eta=0} > \frac{kT}{2nJS}$$

In a previous class we expanded $B_S(\eta)$ about $\eta \ll 1$ and found that

$$B_S(\eta) \simeq \frac{1}{3}(S+1)\eta$$

(in homework you will have to calculate the next term in this expansion)

Then

$$\frac{1}{3}(S+1) > \frac{kT}{2nJS}$$

or $$T < \frac{2nJS(S+1)}{3k} \equiv T_c$$ Curie's temperature.

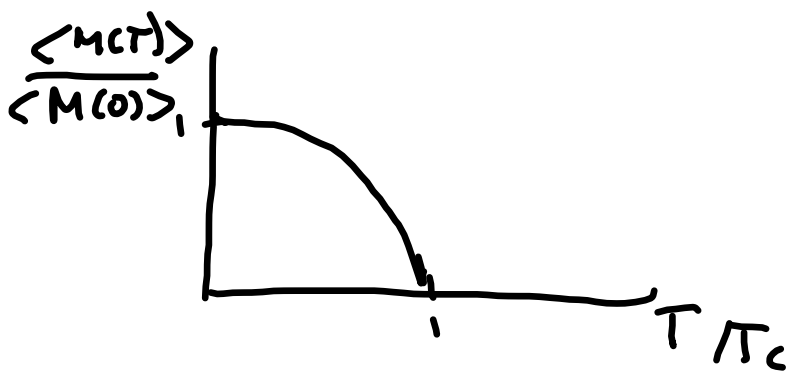
6

If $T < T_c$ then the system is magnetized. (not necessarily fully).

$\uparrow \uparrow \uparrow \uparrow$

If $T > T_c$ the system has $\langle M \rangle = 0$ and it is disordered.

$\uparrow \swarrow \nearrow \downarrow$



For $T \to 0$

$\langle M \rangle \to N g \mu_0 S$

$(B_S \equiv 1)$

Magnetic susceptibility for $H_0 \gtrsim 0$ and $T \gtrsim T_c$.

η is small $\Rightarrow$ $B_s(\eta) \approx \frac{1}{3}(s+1)\eta$

Replacing in (5):

$$\frac{1}{3}(s+1)\eta = \frac{kT}{2nJs}\left(\eta - \frac{g\mu_0 H_0}{kT}\right)$$

$$\eta\left[\frac{kT}{2nJs} - \frac{1}{3}(s+1)\right] = \frac{\cancel{kT}\, g\mu_0 H_0}{2nJs\,\cancel{kT}}$$

$$\eta = \frac{g\mu_0 H_0}{\cancel{2nJs}} \frac{1}{\frac{k}{\cancel{2nJs}}\left(T - \frac{2nJs}{3k}(s+1)\right)} \overset{(6)}{=} \frac{g\mu_0 H_0}{k(T-T_c)} \quad (7)$$

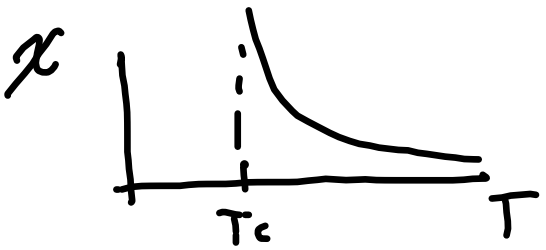
$$M = \frac{1}{3} N g \mu_0 \, s(s+1) \, \eta =$$

$$= \frac{1}{3} N g \mu_0 \, s(s+1) \frac{g \mu_0 H_0}{k(T-T_c)} =$$

$$= \frac{1}{3} \frac{N g^2 \mu_0^2 \, s(s+1)}{k(T-T_c)} H_0$$

$$\chi = \frac{\langle M \rangle}{H_0} = \frac{1}{3} \frac{N g^2 \mu_0^2 \, s(s+1)}{k(T-T_c)}$$

Curie-Weiss law



diverges at T_c.