Second Midterm Exam

P551

November 10, 2016

SHOW ALL YOUR WORK TO GET FULL CREDIT!

WARNING!!! Points will be taken if numerical calculations are nor provided and if calculations are left just indicated.

PART I: **DO IT IN CLASS** Turn your work in before leaving. Take the printed copy of the test home.

PART II: Take the test home and bring **ALL** the questions solved on Tuesday November 15. Your grade for the test will be the **sum of the two** parts. Each question is worth 5 points. A perfect score is worth 100 points as a result of 35 points to be earned in class and 65 points to be earned at home. If you are 100% sure about the work you did in class, you do not need to redo it at home. In that case the points obtained in class will be counted twice.

PART I

Problem 1: A thin-walled vessel of volume V contains a gas at constant temperature T, whose molecules have mass m. The gas, that can be considered ideal, slowly leaks out through a small hole of area A. The outside pressure is low enough that the leakage back into the vessel is negligible. The gas initially contains N_0 molecules.

a) Provide an expression for the number of molecules N_L initially leaking trough the hole per unit time in terms of the temperature and the initial number of molecules of the gas.

b) If the area of the hole is increased by a factor of 3 find the number of molecules that scape per unit time and compare it with the result of (a).

c) If the absolute temperature T of the gas is increased by a factor of 2 while the pressure of the gas is kept constant find the number of molecules that scape per unit time and compare it with the result of (a).

d) If the original gas is replaced by another one whose molecular mass m is 4 times bigger than the one of the original while T, N_0, and the area of the hole A remain the same, find the number of molecules that escape per unit time and compare with the result of (a).

e) If the initial number of molecules in the container were larger than N_0, would the number of molecules that escape per unit time increase, decrease, or stay the same? Why?

f) Under the conditions of part (a) calculate the number of particles that remain in the vessel as a function of time.

g) Under the conditions of part (a) find the time required for the pressure in the vessel to decrease to half its original value.

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PART II

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Problem 2: The plasma at the center of the Sun has a temperature $T = 1.6 \times 10^7$K and it contains a hydrogen density $\rho_H = 6 \times 10^4$kg m^{-3} and a helium density $\rho_{He} = 1 \times 10^5$kg m^{-3}.

a) Obtain the thermal de Broglie wavelength for electrons, protons, and α-particles, i.e. He nuclei, at the center of the Sun.

b) Assuming that the plasma is an ideal gas (no interactions among the particles) determine whether the electron, proton, or *alpha*-particle gases are degenerate in the quantum mechanical sense.

c) Estimate the total gas pressure due to these gas particles near the center of the Sun.

Problem 3: Consider a system of 3 non-interacting spinless fermions.

a) Provide an expression for the partition function $Z_3(\beta) = Tre^{-\beta\hat{H}}$ where $\hat{H}$ is the Hamiltonian matrix. You do not need to perform the integral over the coordinates but you need to write explicitely all the terms. Hint: you can start with Eq.5.5.17.

b) Now consider that the separation between the 3 particles is very large and calculate the integrals explicitely only for the terms that contribute to the lowest correction to the partition function in terms of λ^3/V.

c) Obtain the free energy from the partition function and use it to obtian the equation of state of the "gas". Show that for $\lambda^3/V \ll 1$ you obtain the equation of state for an ideal gas.