11/22

Last time:

Pauli Paramagnetism

$M(T)$ and $\chi(T)$

$$Z_0(N) = \sum_{\{n_p\}} e^{-\beta \sum_p \frac{p^2}{2m} n_p} = e^{-F_0(N)}$$

$$Z(N) = e^{-\beta\mu^* B N} \sum_{N^+=0}^{N} e^{2\beta\mu^* B N^+} Z_0(N^+) Z_0(N-N^+)$$

$$\frac{\ln Z(N)}{N} \simeq -\beta\mu^* B N + \frac{1}{N} \underbrace{e^{2\mu^* B \bar{N}^+ - F_0(\bar{N}^+) - F_0(N-\bar{N}^+)}}_{\text{largest term in the sum obtained from } \frac{\partial}{\partial N} = 0.\ (*)}$$

(*) the equation to obtain $\bar{N}^+$ was:

$$2\mu^* B - \mu_0(\bar{N}^+) + \mu_0(N - \bar{N}^+) = 0 \qquad (1)$$

$$\frac{\partial F_0}{\partial N} = \mu_0$$

Define :

$$M = \mu^*(\bar{N}^+ - \bar{N}^-) = \mu^*(2\bar{N}^+ - N) = \mu^* N r \qquad (1a)$$

$$0 \le r \le 1$$

Then (1) becomes

$$2\mu^* B = \mu_0\left[\frac{(1+r)}{2}N\right] - \mu_0\left[\frac{(1-r)}{2}N\right] \qquad (2)$$

Then:

$$\bar{N}^+ = \frac{Nr + N}{2} = \frac{N}{2}(r+1)$$

$$N - \bar{N}^+ = N - \left(\frac{Nr+N}{2}\right) = \frac{N(1-r)}{2}$$

From ② we see that if $B = 0 \Rightarrow r = 0$

Then $\bar{N}^+ = \bar{N}^- = \frac{N}{2}$ (randomly oriented)

Then to obtain the T dependence when $\mu B \ll kT$ (valid for all T as long as B is small) we can expand ② about $r=0$:

$$2\mu^* B = \underbrace{\left[\mu_0\left(\frac{N}{2}\right) - \mu_0\left(\frac{N}{2}\right)\right]}_{0} + \frac{1}{2}\frac{\partial \mu_0}{\partial x}(xN)\Big|_{x=\frac{1}{2}} r + \frac{1}{2}\frac{\partial \mu_0}{\partial x}(xN)\Big|_{x=\frac{1}{2}} r = \frac{\partial \mu_0(xN)}{\partial x}\Big|_{x=\frac{1}{2}} r$$

Then

$$r \cong \frac{2\mu^* B}{\frac{\partial \mu_0(xN)}{\partial x}\Big|_{x=\frac{1}{2}}}$$

Then:

$$\chi = \frac{M}{VB} \overset{p(1a)}{\simeq} \frac{\mu^* N r}{VB} = \frac{2\mu^{*2} N}{V \frac{\partial \mu_0}{\partial x}(xN)\Big|_{x=\frac{1}{2}}} = \frac{2n\mu^{*2}}{\frac{\partial \mu_0(xN)}{\partial x}\Big|_{x=\frac{1}{2}}} \quad (3)$$

Defining $x = \frac{(1 \pm r)}{2}$ $\quad \frac{\partial x}{\partial r} = \pm\frac{1}{2}$

Notice that (3) is valid for all T as long as B is small.

$\to \mu = kT \ln z$ → found earlier

For $T \to 0$ $\quad \mu \simeq \left(\frac{3N}{4\pi g V}\right)^{2/3} \frac{h^2}{2m}$ $\qquad$ Now $g = 1$, $N \to Nx$

$$\mu_0 = \left(\frac{3xN}{4\pi V}\right)^{2/3} \frac{h^2}{2m}$$

$$\therefore \left.\frac{\partial \mu_0}{\partial x}\right|_{x=\frac{1}{2}} = \frac{2^{4/3}}{3} \left(\frac{3N}{4\pi V}\right)^{2/3} \frac{h^2}{2m}$$

but $\varepsilon_F = \left(\frac{3N}{4\pi 2V}\right)^{2/3} \frac{h^2}{2m}$ $\quad$ (4)

Plugging $\partial\mu_0/\partial x$ in ③ and using ⑦ we obtain:

$$\chi_0 = \frac{2n\mu^{*2}}{\frac{2^{4/3}}{3}\left(\frac{3N}{4\pi V}\right)^{2/3}\frac{h^2}{2m}} = \frac{3}{2}\frac{n\mu^{*2}}{\varepsilon_F}$$ as we already found.

For T low but not zero we use that

$$\mu \simeq \varepsilon_F\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2 + \cdots\right]$$

then

$$\chi \cong \chi_0\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2 + \cdots\right]$$

For $T \to \infty$

$$\mu_0(xN) = kT \ln z = kT \ln\left(\lambda^3 \frac{xN}{V}\right)$$

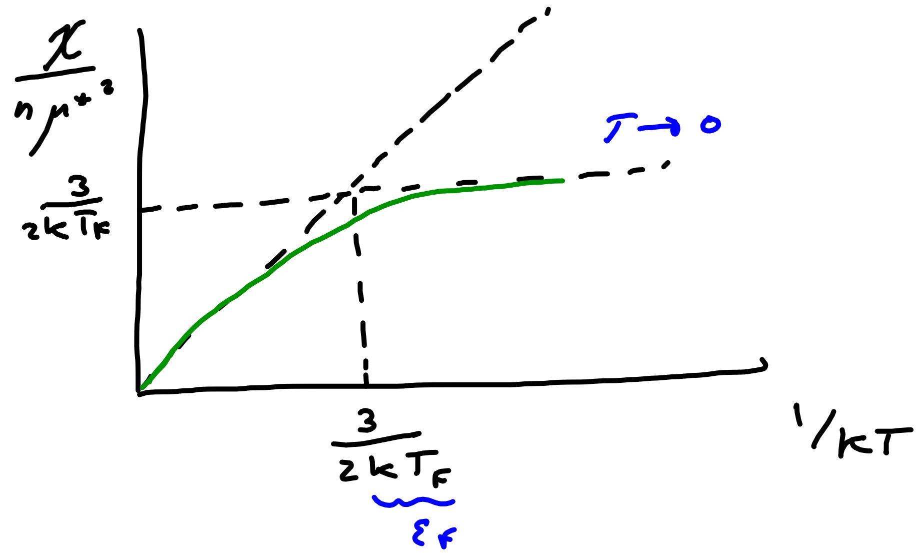
we found

$$\frac{N}{V} = \frac{g}{\lambda^3} f_{3/2}(z) \overset{T\to\infty}{\approx} \frac{z}{\lambda^3} \quad \therefore \quad z = e^{\mu/kT} = \lambda^3 \frac{N}{V} = \lambda^3 n$$

$$\therefore \left.\frac{\partial \mu_0(xN)}{\partial x}\right|_{x=1/2} = \left.\frac{kT \cancel{\lambda^3} \cancel{N/V}}{\cancel{\lambda^3} x \cancel{N/V}}\right|_{x=\frac{1}{2}} = 2kT$$

$$\therefore \chi = \frac{2n\mu^{*2}}{\left.\frac{\partial \mu_0}{\partial x}\right|_{x=1/2}} = \frac{2n\mu^{*2}}{2kT} \quad \text{Curie behavior}$$

Now :

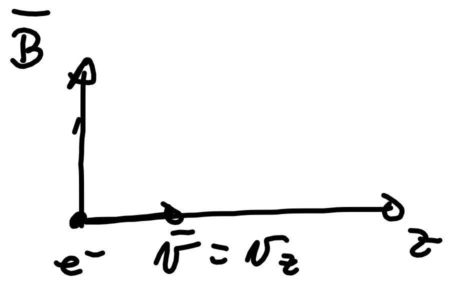


$\frac{\chi}{n\mu^{*2}}$
$\frac{3}{2kT_F}$
$T \to 0$
$\frac{3}{2\underbrace{kT_F}_{\varepsilon_F}}$
$1/kT$

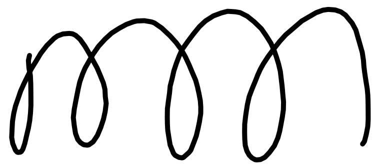
Landau diamagnetism:

Different from Langevin diamagnetism that occurs from the Lenz's law associated to e^- circling a localized ion.

A charged particle moving in a magnetic field describes an spiral motion.



$$\bar{F} = \frac{e}{c}\,\bar{v} \times \bar{B} = -\frac{|e|}{c}\,\bar{v} \times \bar{B}$$



this is the trajectory.

$$F = m\ddot{x} = m\frac{v^2}{r} = \frac{e v B}{c}$$

$$v = \frac{eBr}{cm}$$

$$\frac{v}{2\pi r} = \nu = \frac{eBr}{2\pi r c m} = \frac{eB}{2\pi c m}$$

$$\omega = 2\pi\nu = \frac{eB}{cm}$$

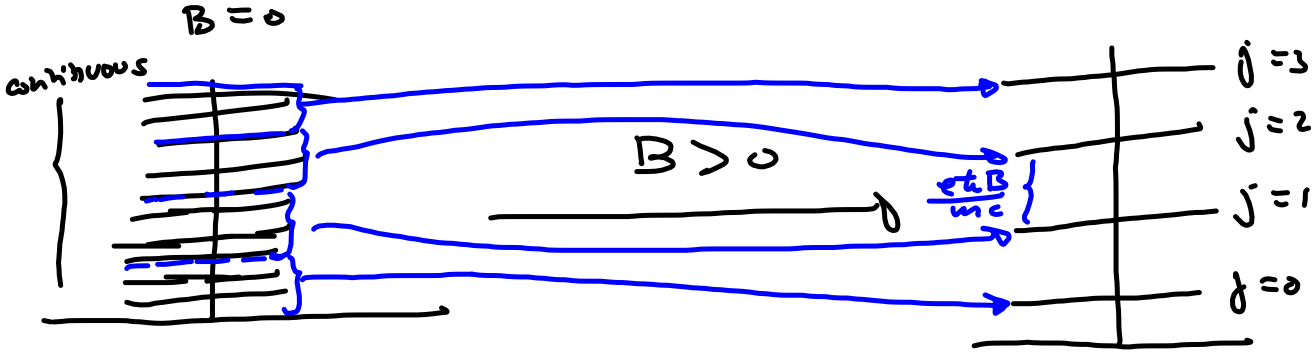
cyclotron frequency independent of r.

Now

$$\epsilon_w = \hbar \omega_c (n + \tfrac{1}{2}) = \hbar \frac{eB}{cm}(n + \tfrac{1}{2})$$

- the energy on the x-y plane is quantized in units of $\frac{eB}{cm}\hbar$
- Along z ϵ_z is also quantized but we are interested in $\Delta \epsilon_z \ll \frac{eB}{cm}$ so we will consider ϵ_z as continuous.

Then: $$\varepsilon = \frac{e\hbar B}{mc}(j + \tfrac{1}{2}) + \frac{p_z^2}{2m} \qquad j = 0, 1, 2, \ldots$$



- The j levels are very degenerate.
- The separation increases with B.
- The degeneracy also increases with B.
- For B very large all the particles are in the j = 0 state.

j's : determine the Landau levels of the gas.

if $\frac{e\hbar B j}{mc} \leq \frac{p_x^2 + p_y^2}{2m} \leq \frac{e\hbar B(j+1)}{mc}$

all states with p in the above range coalesce together.

Then the # of states in the j level is given by:

$$\frac{1}{h^2}\int dx\, dy\, \underbrace{dp_x\, dp_y}_{p\, dp\, d\varphi} = \frac{L_x L_y}{h^2} \underbrace{\int_{\sqrt{\frac{e\hbar B j\, 2m}{mc}}}^{\sqrt{\frac{e\hbar B(j+1)2m}{mc}}} p\, dp}_{\frac{1}{2}p^2} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} =$$

$$= \frac{L_x L_y \pi}{h^2} \, \frac{e \hbar B \, 2\cancel{m}}{\cancel{m} c} (\cancel{j} + 1 - \cancel{j}) = L_x L_y \frac{eB}{hc}$$

independent of j.

Now:

$$\ln \mathcal{Z} = \sum_{\varepsilon} \ln\left(1 + z e^{-\beta \varepsilon}\right) =$$

F.D.

$$= \sum_{j, p_z} \ln\left(1 + z e^{-\beta\left[\frac{e\hbar B}{mc}\left(j + \frac{1}{2}\right) + \frac{p_z^2}{2m}\right]}\right) =$$

$\Delta p_z \ll 1$

$$= \int_{-\infty}^{\infty} \frac{L_z}{h} \, dp_z \left[\sum_{j=0}^{\infty} \left(L_x L_y \frac{eB}{hc} \right) \ln\left(1 + z e^{-\beta\left(\frac{e\hbar B}{mc}\left(j + \frac{1}{2}\right) + \frac{p_z^2}{2m}\right)}\right) \right]$$

Let's consider the case in which $kT \gg \mu^* B$

$\Rightarrow z \ll 1 \Rightarrow$ Boltzmannian.

$$\lim_{\beta \to 0} \ln\left(1 + z e^{-\beta\varepsilon}\right) \approx z e^{-\beta\varepsilon}$$

$\therefore$

$$\ln \mathcal{Z} \approx \int_{-\infty}^{\infty} L_z \frac{dp_z}{h} \left[\sum_{j=0}^{\infty} L_x L_y \frac{eB}{hc}\, z\, e^{-\beta \frac{e\hbar B (j+1)}{mc}} \times e^{-\beta \frac{p_z^2}{2m}} \right] =$$

$$= L_x L_y L_z\, z \frac{eB}{h^2 c} \underbrace{\int_{-\infty}^{\infty} e^{-\beta \frac{p_z^2}{2m}} dp_z}_{\left(\frac{2\pi m}{\beta}\right)^{1/2}} \underbrace{\sum_{j=0}^{\infty} e^{-\beta \frac{e\hbar B (j+\frac{1}{2})}{mc}}}_{\left[2 \sinh\left(\frac{\beta e \hbar B}{2mc}\right)\right]^{-1}}$$

Then

$$\ln \mathcal{Z} \approx \frac{z V e B}{h^2 c} \left(\frac{2\pi m}{\beta}\right)^{1/2} \left[2 \sinh\left(\beta \frac{e \hbar B}{2mc}\right)\right]^{-1}$$

$$\bar{N} = z \frac{\partial \ln \mathcal{Z}}{\partial z}\Big|_{B,V,T} = \frac{zV}{\lambda^3} \frac{x}{\sinh x} \qquad (4a)$$

$$x = \beta \underbrace{\frac{e h}{4\pi m c}}_{\mu_{eff}} B = \beta \mu_{eff} B \qquad (5)$$

Notice that if e and m are charge and mass of e^-
$\mu_{eff} = \mu_B$ but in metals $m = m_{eff} \neq m_e$.

hamiltonian

$$M = \left\langle -\frac{\partial H}{\partial B} \right\rangle = \frac{1}{\beta}\frac{\partial \ln Z}{\partial B}\bigg|_{z,V,T} =$$

$$= \frac{zV}{\lambda^3}\mu_{eff}\left\{ \frac{1}{\sinh x} - x\frac{\cosh x}{\sinh^2 x} \right\} = (5)$$

$$= -\frac{\bar N\, x}{\underset{\mu_{eff}}{\beta B}}\underbrace{\left\{ \coth x - \frac{1}{x} \right\}}_{L(x)} = -\bar N \mu_{eff}\, L(x)$$

Langevin function

diamagnetic response.

This diamagnetic behavior vanishes if $\hbar \to 0$

$\therefore$ it has quantum origins.

If $\mu_{eff} B << kT \Rightarrow x << 1$

$$L(x) \simeq \frac{x}{3} - \frac{x^3}{45} + \cdots \qquad z = \lambda^3 \frac{\bar{N}}{V}$$

4a

$$\bar{N} \simeq \frac{\cancel{3} V}{\lambda^3} \frac{\cancel{x}}{\cancel{x}} = \frac{\cancel{3} V}{\lambda^3} \simeq \lambda^3 \frac{\bar{N}}{V} \frac{V}{\lambda^3} = N$$

\# of particles for $B=0$.

$$M \simeq - \bar{N} \mu_{eff} \frac{x}{3} = - \frac{\bar{N} \mu_{eff}^2 B}{3kT}$$

$x = \beta \mu_{eff} B$

Then

$$\chi_D = \frac{M}{VB} = -\frac{\bar{n}\,\mu_{eff}^2}{3kT}$$ diamagnetic Curie law.

$\therefore$ for an itinerant electron

$$\chi_D = \frac{n\left(\mu_B^2 - \frac{1}{3}\mu_B'^2\right)}{kT}$$

paramagnetic

diamagnetic

$\mu_B' = \mu_{eff}$.

For $\mu_{eff} B << kT$

$$\chi = -\frac{(2\pi m)^{3/2}\mu_{eff}^2 f_{1/2}(z)}{3h^3\beta^{1/2}}$$

(see book 8.2.44 to 8.2.47)

For $z >> 1 \Rightarrow T << T_F \Rightarrow f_{1/2}(z) \approx \frac{2}{\sqrt{\pi}}\sqrt{\ln z}$

$\chi_0 \approx -\frac{1}{2} n \frac{\mu_{eff}^2}{\varepsilon_F} \sim \frac{1}{3}\chi_0$ (paramagnetic).

$\chi_0 \rightarrow$ constant for itinerant electrons.