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Electron gas in metal

F.D statistics allowed to solved some unexplained observed behavior:

1) $C_V = \gamma T + \delta T^3$

γT: due to electrons

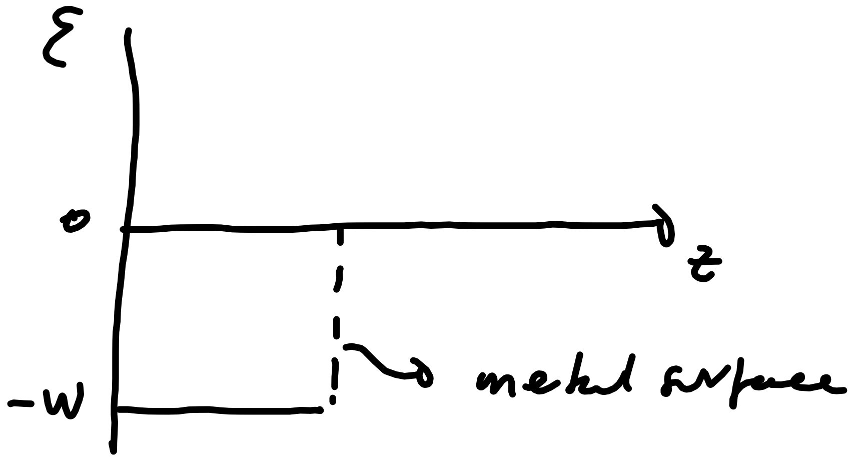
δT^3: due to phonons

instead of $3Nk$ behavior expected from equipartition.

2. Thermionic emission.

e^- do not come out spontaneously from metals at room temperature. However, if T is raised a thermionic current develops.

This is because the ions in the metal produce a bonding energy W.



e^- has to have $\mathcal{E} > W$ to be able to come out

since $\mathcal{E} = \frac{p^2}{2m}$

then in this case we need $\frac{p_z^2}{2m} > W$

To find out how many e^- leave per unit area and per unit time we calculate R as we did for effusion:

$$R = \int_{p_z=\sqrt{2mW}}^{\infty} dp_z \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \, \frac{\overset{\nearrow \text{spins}}{2}}{h^3} \, u_z \, \underbrace{\frac{1}{e^{\frac{(\varepsilon-\mu)}{kT}}+1}}_{\text{F-D instead of M-B distribution}} =$$

$\varepsilon = \frac{p^2}{2m} = \frac{p'^2+p_z^2}{2m}$ $\quad p_x^2+p_y^2 = p'^2 \quad \tan\theta = \frac{p_y}{p_x}$

$$= \frac{2}{h^3} \int_{\sqrt{2mW}}^{\infty} dp_z \, u_z \int_0^{\infty} p' \, dp' \frac{1}{e^{\frac{p'^2+p_z^2}{2mkT}} e^{-\frac{\mu}{kT}}+1} \int_0^{2\pi} d\theta =$$

$u_z = \frac{p_z}{m}$

$\varepsilon_z = \frac{p_z^2}{2m}$

$$= \frac{4\pi}{h^3} \int_{\sqrt{2mW}}^{\infty} \frac{p_z}{m} \, dp_z \, \ln\left[1+e^{\left[\mu-\frac{p_z^2}{2m}\right]\beta}\right] =$$

$$= \frac{4\pi m kT}{h^3} \int_{\varepsilon_z = W}^{\infty} d\varepsilon_z \ln\left(1 + e^{\beta(\mu - \varepsilon_z)}\right)$$

Notice that

$$e^{\frac{\mu - \varepsilon_z}{kT}} << 1 \quad \forall T \Rightarrow \ln(1+x) \approx x$$

$$\therefore \ln\left(1+e^{\beta(\mu-\varepsilon_z)}\right) \approx e^{\beta(\mu - \varepsilon_z)}$$

$$-kTe^{\frac{\mu-\varepsilon_z}{kT}}\Big|_W^{\infty} = kTe^{\frac{\mu - W}{kT}}$$

$\therefore$

$$R = \frac{4\pi m kT}{h^3} \underbrace{\int_{\varepsilon_z = W}^{\infty} e^{\frac{\mu - \varepsilon_z}{kT}} d\varepsilon_z}_{kTe^{\frac{\mu - W}{kT}}} = \frac{4\pi m k^2 T^2}{h^3} e^{\frac{(\mu - W)}{kT}}$$

Then the thermionic current J is:

$$J = eR = \frac{4\pi m e k^2 T^2}{h^3} e^{\frac{\mu - W}{kT}}$$

increases with T from almost 0 at room temperature.

This result is in good agreement with experiments.

Classically $J_{class} \neq J$:

$$\frac{1}{e^{\frac{(\varepsilon-\mu)}{kT}}+1} \xrightarrow{\text{class}} e^{-\frac{(\varepsilon-\mu)}{kT}} \qquad M-B.$$

Replacing in $\mathcal{R}$:

$$\mathcal{R} = \frac{4\pi}{h^3}\int_{p_z=\sqrt{2mW}}^{\infty} \frac{p_z}{m}\,dp_z \underbrace{\int_0^{\infty} p' e^{-\frac{p'^2}{2mkT}}\,dp'}_{\frac{2mkT}{2}\underbrace{\Gamma(1)}_{1}}\; e^{-\frac{p_z^2}{2mkT}+\frac{\mu}{kT}} =$$

$$= \frac{4\pi kT}{h^3}\underbrace{e^{\mu/kT}}_{\substack{z \\ n\,\text{''}\,\lambda^3/g}}\int_{\sqrt{2mW}}^{\infty} p_z e^{-p_z^2/2mkT}\,dp_z\;\frac{2mkT}{2} =$$

$$\varepsilon_z = p_z^2/2m$$

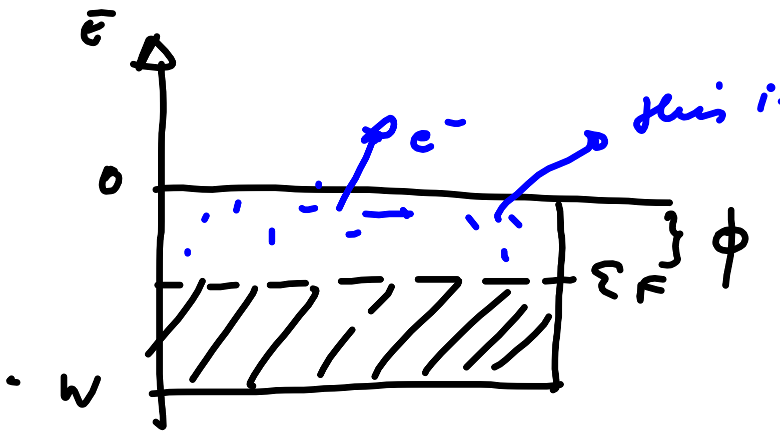
$$= \frac{4\pi k T n h^3 m}{2\left(\sqrt{2\pi m k T}\right)^3} \underbrace{\int_W^\infty d\varepsilon_z \, e^{-\frac{\varepsilon_z}{kT}}}_{-kT e^{-\varepsilon_z/kT}\Big|_W^\infty = kT e^{-W/kT}} =$$

$$= \frac{2\pi k T n m k T}{(2\pi m k T)^{3/2}} e^{-W/kT}$$

$$J_{class} = eR = ne\left(\frac{k}{2\pi m}\right)^{1/2} T^{1/2} e^{-\phi/kT} \qquad \phi = W$$

$$J_{F.D} = \frac{4\pi m e k^2}{h^3} T^2 e^{-\phi/kT} \qquad \phi = W - \varepsilon_F$$

work function of the metal.



this is populated with e^- as T increases.

Photoelectric emission:

Even at room temperature e^- can be made come out by providing extra energy. One way is via a photon beam.

Classical paradox: $J \neq 0$ only if ν of light was larger than ν_0 (a threshold frequency). People expected that ν would not play a role and that the intensity of the beam would matter. But experimentally it was irrelevant.

Using quantum mechanics we know that to remove an electron we need:

$$\frac{p_z^2}{2m} + h\nu > W$$

$$R = \int_{p_z=\sqrt{(W-h\nu)2m}}^{\infty} dp_z \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \, \frac{2}{h^3} \frac{1}{e^{\frac{\varepsilon-\mu}{kT}}+1} u_z =$$

$$= \frac{4\pi m k T}{h^3} \int_{\varepsilon_z = W-h\nu}^{\infty} d\varepsilon_z \, \ln\left[1 + \underbrace{e^{\frac{\mu-\varepsilon_z}{kT}}}\right]$$

now T is not high enough so that $e^{(\,)} \ll 1$.

Define:

$$x = \frac{\varepsilon_z - W + h\nu}{kT} \Rightarrow dx = \frac{d\varepsilon_z}{kT}$$

$$R = \frac{4\pi m (kT)^2}{h^3}\int_0^\infty dx\, \ln\left(1 + e^{\frac{\mu - kTx - W + h\nu}{kT}}\right) =$$

$$= \frac{4\pi m (kT)^2}{h^3}\int_0^\infty dx\, \ln\left[1 + e^{\frac{\mu - W}{kT} + \frac{h\nu}{kT} - x}\right]$$

Define $h\nu_0 = W - \mu \simeq W - \varepsilon_F = \phi$

$$R = \frac{4\pi m (kT)^2}{h^3}\int_0^\infty dx\, \ln\left[1 + e^{\frac{h(\nu - \nu_0)}{kT} - x}\right]$$

$\nu_0 = \frac{\phi}{h}$ is the threshold frequency. If $\nu < \nu_0$ no e^- come out.

$$J = R_e = \frac{4\pi m_e k^2 T^2}{h^3} \int_0^\infty dx \, \ln \underbrace{\left[1 + e^{\delta - x}\right]}_{u} \underbrace{1}_{v'} =$$

$$\delta = \frac{h(\nu - \nu_0)}{kT}$$

$$= \frac{4\pi m_e k^2 T^2}{h^3} \left[\underbrace{x \ln(1 + e^{\delta - x}) \Big|_0^\infty}_{0} - \int_0^\infty \frac{x(-e^{\delta - x})}{1 + e^{\delta - x}} dx \right]$$

$$= \frac{4\pi m_e k^2 T^2}{h^3} \underbrace{\int_0^\infty \frac{x}{e^{x-\delta} + 1} dx}_{f_2(e^\delta)} = \frac{4\pi m_e k^2 T^2}{h^3} f_2(e^\delta)$$

If $h(\nu-\nu_0) >> kT \Rightarrow e^{\delta} >> 1$

$$\therefore f_2(e^{\delta}) \approx \frac{\delta^2}{2}$$

$$\therefore J \approx \frac{2\pi m e}{h}(\nu-\nu_0)^2$$

independent of T
(you have very energetic photons)

if $h\nu >> \phi$ T is irrelevant.

If $\nu < \nu_0$ and $h|\nu-\nu_0| >> kT$ $e^{\delta} << 1$

$\therefore f_2(e^{\delta}) \approx e^{\delta}$ and

$$J \approx \frac{4\pi m e k^2 T^2}{h^3} e^{(h\nu-\phi)/kT}$$

thermionic current with reduced effective $\phi' = \phi - h\nu$

If $\nu = \nu_0 \Rightarrow \delta = 0 \quad \therefore \quad f_2(1) = \dfrac{\pi^2}{12}$

$$\therefore \quad J_0 = \frac{\pi^3 m e k^2 T^2}{3 h^3}.$$

Work function: