Homework #2

Problem 5:

Consider the problem of a classical particle of mass m confined to move in a one-dimensional box of length L. Consider that the particle's energy E satisfies $E_0 - \delta/2 < E < E_0 + \delta/2$.

a)*i*) Find the lowest absolute value that the particle's momentum can have in terms of E_0 and δ and call this value p_0.

ii) Find the highest absolute value that the particle's momentum can have in terms of E_0 and δ and call this value $p_0 + \Delta$.

Now assume that $\Delta = p_0/10$.

b) What is the macrostate for this situation? Provide the values of E, N and V that define the macrostate and draw the regions in phase space consistent with the macrostate.

c) Now assume that the unit of volume in phase space is given by $\omega_0 = L/10 \times \Delta/4$, i.e., phase space should be devided into cells of length $L/10$ along the q axis and $\Delta/4$ along the p axis. Make a draw and say how many cells are inside the allowed regions of phase space. Notice that while technically there are infinite values of (q, p) inside each cell of volume ω_0 for practical meassurements all these states are the same and each cell corresponds to one physical microstate at the classical level, while quantum mechanics will do this for you.

d) Now consider the "volume" $\Delta\omega$ defined by the points $A = (0, p_0)$, $B = (0, p_0+\Delta)$, $C = (\epsilon, p_0)$, and $D = (\epsilon, p_0+\Delta)$ with $\epsilon = 2L/10$. Draw $\Delta\omega$ in the phase space diagram and say how many cells of volume ω_0 are in it.

e) Now calculate T_A, the time that takes the particle to reach $q = L$ when it starts at point A for t=0.

f) Calculate the position in phase space of points A, B, C, and D at time $t = T_A/2$ and draw the volume $\Delta\omega$ in phase space at this time.

i) Has the shape change?

ii) Has the "volume change"?

iii) How many ω_0 cells fit inside $\Delta\omega$? How does this compare with the number of cells inside $\Delta\omega$ at $t = 0$?

g) Calculate the position in phase space of points A, B, C, and D at time $t = T_A$ and draw the volume $\Delta\omega$ in phase space at this time.

i) Has the shape change?

ii) Has the "volume change"?

iii) How many ω_0 cells fit inside $\Delta\omega$? How does this compare with the number of cells inside $\Delta\omega$ at $t = 0$ and at $t = T_A/2$?

h) Calculate the position in phase space of points A, B, C, and D at time $t = 10T_A$ and draw the volume $\Delta\omega$ in phase space at this time.

i) Has the shape change?

ii) Has the "volume change"?

iii) How many ω_0 cells fit inside $\Delta\omega$? How does this compare with the number of cells inside $\Delta\omega$ at $t = 0$, $t = T_A/2$, and $t = T_A$?

i) Calculate the position in phase space of points A, B, C, and D at time $t = 100T_A$ and draw the volume $\Delta\omega$ in phase space at this time.

i) Has the shape change?

ii) Has the "volume change"?

iii) How many ω_0 cells fit inside $\Delta\omega$? How does this compare with the number of cells inside $\Delta\omega$ at $t = 0$, $t = T_A/2$, $t = T_A$, and $t = 10T_A$?

j) What do you think will happen as $t \to \infty$?

This problem should give you an idea of how entropy can increase in an irreversible problem without violating Liouville's theorem. Imagine that at $t = 0$ the phase space accessible to the particle is just $\Delta\omega$. The entropy is given by the number of cells inside $\Delta\omega$ at $t = 0$. As time goes by, $\Delta\omega$ evolves eventually covering all the cells in the available phase space, indicating an increase in entropy while its volume remains constant!