First Midterm Exam

P551
September 28, 2017

SHOW ALL WORK TO GET FULL CREDIT!
WARNING!!! Points will be taken if numerical calculations are nor provided and if calculations are left just indicated.

PART I: **DO IT IN CLASS** Turn your work in before leaving. Take the printed copy of the test home.

PART II: Take the test home and bring **ALL** the questions solved on Tuesday, October 3. Your grade for the test will be the **sum of the two** parts. A perfect score is worth 200 points as a result of 50 points to be earned in class and 150 points to be earned at home. If you are 100% sure about the work you did in class, you do not need to redo it at home. In that case the points obtained in class will be counted twice.

PART I

Problem 1: Consider a 1D system in which two indistinguishable particles can occupy six different sites separated by a distance a_0 from each other (see figure). Only one of the two particles can occupy a given site. The energy of the system depends on the separation between the two particles and is given by $E(r) = -\frac{\epsilon}{(r/a_0)^2}$.

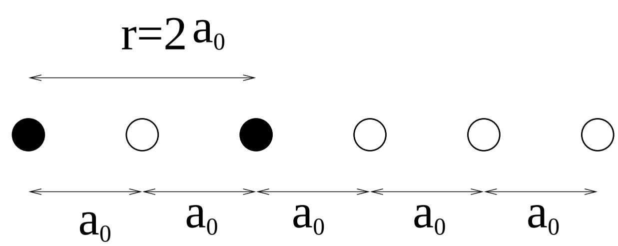

FIG. 1:

a) Find all the energy levels of this system and provide the degeneracy of each one. (5 points)

b) Write the canonical partition function for the system. (5 points)

c) Provide an expression for the average energy. (5 points)

d) Provide an expression for the average separation between the particles $\langle r \rangle$. (5 points)

e) In what state will the system be when $T = 0$? (3 points) Why? (2 points) Provide the energy of the system, in terms of ϵ, for $T = 0$. (5 points)

f) Provide the energy of the system, in terms of ϵ, for $T \to \infty$. (5 points)

g) For $T = (\epsilon/k)$ provide the probability of finding the system in each of the energy levels that you found in (a). (5 points)

h) Provide the probability of finding the system in each of the energy levels that you found in (a) for $T = 0$. (5 points)

i) Provide the probability of finding the system in each of the energy levels that you found in (a) for $T \to \infty$. (5 points)

STOP HERE!!!!: Hand your work before leaving and take home the printed copy of the test. Bring **ALL** the questions answered on Tuesday, October 3.

PART II

This is the continuation of Problem 1. You should redo questions (a) to (i) again at home unless you feel totally confident about your in-class work. If you choose NOT to redo questions (a) to (i) your work in class will be counted twice for your final grade. By redoing the problem at home you have a chance of getting a higher grade.

j) Provide the average energy of the system, in terms of ϵ, for $T = \epsilon/k$. (5 points)

k) Make a plot of $< E > /\epsilon$ versus $T/(\epsilon/k)$. Hint: it will be best if you use a plotting program to do the plot and verify the results that you obtained in (e), (f) and (j). If you chose to make the plot by hand you need to show the results of part (e), (f) and (j) plus two additional points of your choice. (10 points)

l) Provide the average separation between particles $\langle r \rangle$, in terms of a_0, for $T \to \infty$. (5 points)

m) Provide the average separation between particles $\langle r \rangle$, in terms of a_0, for $T = 0$. (5 points)

n) Provide the average separation between particles $\langle r \rangle$, in terms of a_0, for $T = (\epsilon/k)$. (5 points)

o) Make a plot of $< r > /a_0$ versus $T/(\epsilon/k)$. Hint: it will be best if you use a plotting program to do the plot and verify the results that you obtained in (l), (m) and (n). If you chose to make the plot by hand you need to show the results of part (l), (m) and (n) plus two additional points of your choice. (10 points)

Problem 2: Consider a particle of mass $m = 9.31 \times 10^8$ eV/c^2 which moves without interactions inside a one-dimensional "box " of length $L = 1m$. Here $c = 3 \times 10^8$m/s is the speed of light. The particle's energy is in the range $(E, E + \Delta E)$ with $E = 13$ meV and $\Delta E = 3$ meV.

a) Draw the regions in phase space accessible to the particle and provide numerical values for the boundaries of the accessible region. Hint: be careful with the units since $[m] = eV/c^2$, $[p] = eV/c$. (5 points)

b) Assume that the unit of volume in phase space is given by $\omega_0 = h = 4.1357 \times 10^{-15}$ eV.s and that $\delta x = 2 \times 10^{-3}$m. Provide the numerical value of δp. (3 points) Provide the ratio $\Delta p/\delta p$ where Δp is the width of the accessible phase space along the p axis found in (a). (2 points)

c) Find the number of accessible states Ω_c. (5 points)

d) Find the entropy S of the system and provide its numerical value. Hint: $k = 8.62 \times 10^{-5}$ eV/K. (5 points)

e) Now, using the quantum mechanical expression for the energy levels of a particle in a 1D box given by $E_n = \frac{n^2h^2}{8mL^2}$ determine the value of n associated with E and $E + \Delta E$ whose numerical values were provided in part (a). (5 points)

f) Now find the dimensions of the unit of volume ω_0' in phase space using quantum mechanics. Provide numerical values for its length, $\delta' x$ and $\delta' p$, along the axis in phase space. Provide the ratio $\Delta p/\delta' p$ where Δp is the width of the accessible phase space along the p axis found in (a) and compare with the result obtained in (b) for $\Delta p/\delta p$. Hint: Find $\delta' E$, the minimum energy interval quantum mechanically allowed for $E = 13meV$ and then you can calculate $\delta' p$ using $\delta' p = \frac{dp}{dE}\delta' E$. (10 points)

g) Find the number of accessible states Ω_q using the results of (f) and compare with the result of part (c). (5 points) Would you expect Ω_c and Ω_q to be equal or different from each other? (2 points) Why? (3 points)

h) Now assume that E given in (a) is the mean energy of the system and calculate the temperature T of the system providing its value in K (degrees Kelvin). Hint: These integrals will be helpful:$\int_0^\infty x^n e^{-ax^2} dx = \frac{1}{2}\Gamma(\frac{n+1}{2})/a^{\frac{n+1}{2}}$ for $n > -1$ and $a > 0$ with $\Gamma(1/2) = \sqrt{\pi}$ and $\Gamma(n + \frac{1}{2}) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$ for $n = 1, 2, ...$). (10 points)

i) Now assume that we put N particles inside the 1D box and the temperature of the system remains at the same value that you found in (h). Write the partition function of the system of N particles and provide the value of its average total energy in eV in terms of N and compare with the value of E provided in the first paragraph of the problem. (5 points)