First Midterm Exam

P551
September 27, 2018

SHOW ALL WORK TO GET FULL CREDIT!
WARNING!!! Points will be taken if numerical calculations are nor provided and if calculations are left just indicated.

PART I: **DO IT IN CLASS** Turn your work in before leaving. Take the printed copy of the test home.

PART II: Take the test home and bring **ALL** the questions solved on Tuesday, October 2. Your grade for the test will be the **sum of the two** parts. A perfect score is worth 170 points as a result of 50 points to be earned in class and 120 points to be earned at home. There are two bonus questions worth 5 points each in the at home part. If you are 100% sure about the work you did in class, you do not need to redo it at home. In that case the points obtained in class will be counted twice.

PART I

Problem 1: Consider a polymer formed by N connecting disk-shaped molecules forming a one-dimensional chain. Each molecule can align along either its long axis (of length $2a$) or short axis (length a). The energy of the molecule aligned along its shorter axis is higher by ϵ, that is, the total energy $E = \epsilon U$, where U is the number of molecules standing up.

a) What is the total number of microstates that the polymer has? Give your result in terms of N. (5 points)

b) If the polymer has a total energy $E = N\epsilon/4$ how many microstates are accessible to the system? Provide your result in terms of N. (5 points)

c) What is the entropy of the system? (5 points)

d) What is the length of the polymer? (5 points)

e) What is the canonical partition function for the polymer? (5 points)

Problem 2: Consider N non-interacting particles of mass m confined to a two dimensional square box with sides of length L. The total energy of the system is in the interval $(E, E + \Delta E)$.

a) What is the dimension of the system's phase space? (5 points)

b) Consider the x and y coordinate of particle 1 and the x coordinate of particle 2 and draw the accessible region in phase space expanded by these 3 coordinates. Use the data provided to label your graph.(5 points)

c) Consider the x and y components of the momentum of particle 1 and the x component of the momentum of particle 2 and draw the accessible region in phase space expanded by these 3 components. Use the data provided to label your graph.(5 points)

d) Calculate the total number of microstates allowed for the system assuming that the particles are distinguishable. Give the answer in terms of L, N, E, and Δ. Hint: the volume of a sphere of radius R in dimension n is given by $V_n(R) = \frac{\pi^{n/2} R^n}{(n/2)!}$ and that the unit of volume in phase space is $h^{\mathcal{N}}$ where $\mathcal{N}$ is the number of degrees of freedom of the gas. (5 points)

e) If now we assume that the particles are indistinguishable, how does the number of microstates change and why? (5 points)

STOP HERE!!!!: Hand your work before leaving and take home the printed copy of the test. Bring **ALL** the questions answered on Tuesday, October 2.

PART II

You should redo questions (a) to (e) again at home (for the two problems) unless you feel totally confident about your in-class work. If you choose NOT to redo questions (a) to (e) your work in class will be counted twice for your final grade. By redoing the questions at home you have a chance of getting a higher grade. Each question is worth 5 points, including the two bonus questions.

Problem 1: Consider a polymer formed by N connecting disk-shaped molecules forming a one-dimensional chain. Each molecule can align along either its long axis (of length $2a$) or short axis (length a). The energy of the molecule aligned along its shorter axis is higher by ϵ, that is, the total energy $E = \epsilon U$, where U is the number of molecules standing up.

a) What is the total number of microstates that the polymer has? Give your result in terms of N.

b) If the polymer has a total energy $E = N\epsilon/4$ how many microstates are accessible to the system? Provide your result in terms of N.

c) What is the entropy of the system?

d) What is the length of the polymer?

e) What is the canonical partition function for the polymer?

f) Now assume that we know that the average energy of the polymer is $< E >= N\epsilon/4$. Find the temperature of the system. Provide your answer in terms of ϵ.

g) Now that you know the partition function of the system provide an expression for its free energy F.

h) Using the results of part (f) and (g) calculate the entropy of the polymer when it is at the temperature you found in (f).

i) Compare the result you found in part (h) with the result you found in part (c) and explain whether the result of the comparison is what you expected and why.

j) What is the average length of the polymer at the temperature you found in part (f)? How does it compare with the length you found in part (d)?

k) Find the average energy of the system when $T \to \infty$. Provide the result in terms of N and ϵ.

l) If the system were prepared with the energy you found in part (k) how many microstates would be accessible to the system?

m) What is the entropy of the system in this case?

n) How many molecules are standing up when $t \to \infty$?

o) Bonus question: Can the length of the polymer be equal to Na? Why?

Problem 2: Consider N non-interacting particles of mass m confined to a two dimensional square box with sides of length L. The total energy of the system is in the interval $(E, E + \Delta E)$.

a) What is the dimension of the system's phase space?

b) Consider the x and y coordinate of particle 1 and the x coordinate of particle 2 and draw the accessible region in phase space expanded by these 3 coordinates. Use the data provided to label your graph.

c) Consider the x and y components of the momentum of particle 1 and the x component of the momentum of particle 2 and draw the accessible region in phase space expanded by these 3 components. Use the data provided to label your graph.

d) Calculate the total number of microstates allowed for the system assuming that the particles are distinguishable. Give the answer in terms of L, N, E, and Δ. Hint: the volume of a sphere of radius R in dimension n is given by $V_n(R) = \frac{\pi^{n/2} R^n}{(n/2)!}$ and that the unit of volume in phase space is $h^{\mathcal{N}}$ where $\mathcal{N}$ is the number of degrees of freedom of the gas.

e) If now we assume that the particles are indistinguishable, how does the number of microstates change and why?

f) Provide the entropy of the gas made of indistinguishable particles.

g) Provide the energy of the gas in terms of its temperature. How does it compare with the energy of a gas in 3D?

h) Calculate the partition function for the gas made of indistinguishable particles.

i) Calculate the gas entropy from the partition function and compare your answer with your result of part (f).

j) Find the equation of state for the 2D gas and compare it with the equation of state for the gas in 3D.

k) Bonus: Obtain $g(E)$, the density of states for the gas and compare with the result for the 3D gas.