Problem 4:

We want to write the continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0, \]

in covariant form knowing that \( J^\mu = (c\rho, \mathbf{J}) \) is a 4-vector.

We know that the covariant derivative is given by

\[ \partial_\alpha = \left( \frac{\partial}{\partial t}, \nabla \right). \]

We see that

\[ \partial_\alpha J^\alpha = \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J}. \]

Then, the covariant form of Eq.(1) is

\[ \partial_\alpha J^\alpha = 0. \]