From Homework problem (1.4) (b) we know that the electric field inside a uniformly charged sphere is:

\[ E_{\text{inside}} = \frac{Qr}{4\pi\varepsilon_0 R^3} \]

if \( R \) is the radius and it points along \( \hat{r} \). At the surface \( r = R \):

\[ \mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \hat{r} \]

where \( \hat{r} = \sin\theta \cos\phi \hat{x} + \sin\theta \sin\phi \hat{y} + \cos\theta \hat{z} \)

By symmetry, the net force is along the \( z \) axis. Then, only we can about the \( z \) component of the Maxwell stress tensor:

\[ \sum T_{\alpha\beta} \eta_{\beta\gamma} = \left( T_{2x} M_x + T_{2y} M_y + T_{2z} M_z \right) \]

\[ T_{2x} = E_0 E_z E_x = E_0 \left( \frac{Q}{4\pi\varepsilon_0 R^2} \right)^2 \cos\theta \sin\theta \cos\phi \]

\[ T_{2y} = E_0 E_z E_y = E_0 \left( \frac{Q}{4\pi\varepsilon_0 R^2} \right)^2 \cos\theta \sin\theta \sin\phi \]

\[ T_{2z} = \frac{E_0}{2} (E_z^2 - E_x^2 - E_y^2) = \frac{E_0}{2} \left( \frac{Q}{4\pi\varepsilon_0 R^2} \right)^2 \left( \cos^2\theta - \sin^2\theta \cos^2\phi \right) \left( \cos^2\phi - \sin^2\phi \right) \]
\[ \eta_x = \sin \theta \cos \phi \]
\[ \eta_y = \sin \theta \sin \phi \]
\[ \eta_z = \cos \theta \]

\[ \sum \eta_z M_y = E_0 \left( \frac{Q}{4 \pi \varepsilon_0 R^2} \right)^2 \left[ \frac{\cos \theta \sin^2 \theta \cos^2 \phi + \cos \theta \sin^2 \theta \sin^2 \phi}{\cos \theta \sin^2 \theta + \cos \theta \frac{r^2}{2} - \sin^2 \theta \cos \theta} \right] \]

\[ = \frac{E_0}{2} \left( \frac{Q}{4 \pi \varepsilon_0 R^2} \right)^2 \cos \theta \]

\[ da = R^2 \sin \theta \, d\phi \, d\theta \]

Then:

\[ \sum \eta_z M_y \, da = \frac{E_0}{2} \left( \frac{Q}{4 \pi \varepsilon_0 R} \right)^2 \cos \theta \sin \theta \, d\phi \, d\theta \]

Integrating over the upper surface:

\[ \frac{1}{2} \iint \sum \eta_z M_y \, da = \frac{E_0}{2} \left( \frac{Q}{4 \pi \varepsilon_0 R} \right)^2 \left[ \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos \theta \sin \theta \, d\theta \right] \]

\[ = \frac{E_0}{2} \left( \frac{Q}{4 \pi \varepsilon_0 R^2} \right)^2 \frac{2\pi}{4} \frac{1}{2} \]

\[ \left[ \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q^2}{8R^2} \right] \]
This is the force on the "bowl"

We still need to calculate for the lower disk ($\theta = \pi/2$)

Here $M_x = M_y = 0$, $M_z = -1$

$$d\mathbf{a} = (r \, d\phi) \, dr \, \hat{e}_z$$

From the values of $M_z$, we need only $T_{zz}$

$$\int_{\text{lower disk}} T_{zz} \, M_z \, d\mathbf{a} = \frac{E_0}{2} \left( E_z^2 - E_x^2 - E_y^2 \right) r \, d\phi \, dr$$

From Homework problem (1.4) (b)

$$|E_{\text{inside}}| = \frac{Q_r}{4\pi E_0 R^3}$$

pointing along $\hat{e}_r$

Since $\theta = \pi/2$, $\hat{e}_r = \cos \phi \, \hat{e}_x + \sin \phi \, \hat{e}_y$

Then, $E_z$ in lower disk = 0 (reasonable by symmetry)

$$E_x = \frac{Q_r}{4\pi E_0 R^3} \cos \phi$$

$$E_y = \frac{Q_r}{4\pi E_0 R^3} \sin \phi$$
\[ E_z - E_x = \frac{Q^2}{4\pi \varepsilon_0 R^3} (0 - \cos^2 \phi - \sin^2 \phi) \]

\[ \int T_{zz} m_2 \, da = \frac{\varepsilon_0}{2} \int_0^{2\pi} d\phi \int_0^R r^2 \, dr \frac{Q^2 r^2}{(4\pi \varepsilon_0 R^3)^2} (-1) (-1) \]

lower disk

\[ = \frac{\varepsilon_0}{2} \frac{Q^2}{(4\pi \varepsilon_0 R^3)^2} \int_0^R r^3 \, dr = \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{16 R^2} \]

This points along the z-axis, in the positive direction (it is also repulsive).

Total Force = \[ \int_{\text{all surface}} \sum_\beta E_{zz} m_\beta \, da = \frac{1}{4\pi \varepsilon_0} \frac{3 Q^2}{16 R^2} \]

Note that \( \frac{Q}{\partial t} \) field = 0 since \( \vec{E} \) is t-independent.

So this force is compensated by some mechanical device that keeps the charge in place, otherwise it would basically explode.
As Griffiths explains, we could use other geometries. For instance:

The contribution at \( R \to \infty \) is 0 because \( \mathbf{E} \to 0 \) as \( R \to \infty \).

With regards to \( z = 0 \), we already did the shaded circle. So now we have to handle the rest.

The electric field in that region is that of a point like particle at the origin, with charge \( Q \),

\[
|\mathbf{E}| = \frac{Q}{4\pi \varepsilon_0 R^2} \quad \text{pointing away from the origin.}
\]

\[
\mathbf{E}_r = \cos \phi \hat{\mathbf{r}} + \sin \phi \hat{\mathbf{\theta}} \quad \text{as before (} \theta = \pi / 2\text{)}
\]

\[
E_x = \frac{Q}{4\pi \varepsilon_0 R^2} \cos \phi, \quad E_y = \frac{Q}{4\pi \varepsilon_0 R^2} \sin \phi
\]

We only need \( T_{zz} \) (by symmetry)

\[
T_{zz} = \frac{\varepsilon_0}{2} \left( E_z^2 - \frac{1}{2} (E_x^2 + E_y^2) \right) = \frac{\varepsilon_0}{2} (E_x^2 + E_y^2)
\]

\[
= - \frac{\varepsilon_0}{2} \left( \frac{Q}{4\pi \varepsilon_0 R^2} \right)^2 \left( \cos^2 \phi + \sin^2 \phi \right) = -1
\]

\[
da = rd\phi \, dr \quad \text{pointing in the } (-k) \text{ direction.}
Then \[ \oint \sum_{n \neq 0} T_{2n} \psi \varphi \, da = \]

\[ = \int_{0}^{2\pi} \int_{0}^{\infty} \frac{e_{0}}{2} \left( \frac{Q}{\epsilon_{0} \pi R} \right)^{2} \frac{1}{r^{4}} dr \, d\phi \]

\[ = \frac{1}{4\pi \epsilon_{0}} \cdot \frac{Q^{2}}{2} \epsilon_{0} \int_{0}^{\infty} \frac{dr}{r^{3}} \int_{0}^{2\pi} \frac{d\phi}{2} = \frac{1}{4\pi \epsilon_{0}} \cdot \frac{Q^{2}}{8 \pi R^{2}} \]

which is the same value as for the "bowl"