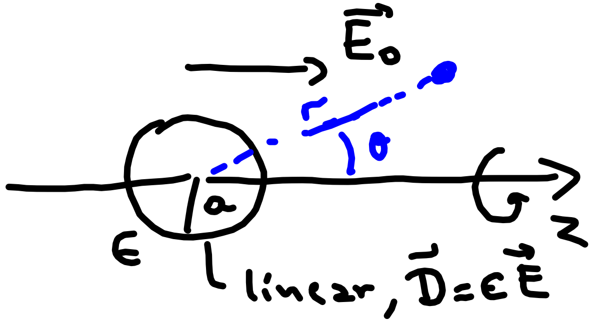
Example 2 (page 157)



Inside: $$\phi = \sum_{l=0}^{\infty} \underline{\underline{A_l}}\, r^l P_l(\cos\theta)$$

to be found

Outside: $$\phi = -E_0 \underbrace{r\cos\theta}_{P_1(\cos\theta)} + \sum_{l=0}^{\infty} \underline{C_l}\, r^{-(l+1)} P_l(\cos\theta)$$

$l = 1$ diff. $l \neq 1$

Boundary conditions

$\sigma = 0$

$$\vec{E} = -\nabla\phi$$
$$\hookrightarrow \hat{e}_\theta \frac{1}{r}\frac{\partial}{\partial\theta} + \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\phi \cdots$$

From

$$(\vec{E}_2 - \vec{E}_1) \times \vec{n}_{21} = 0 \implies \left. -\frac{1}{r}\frac{\partial \phi_{in}}{\partial \theta}\right|_{r=a} = \left. -\frac{1}{r}\frac{\partial \phi_{out}}{\partial \theta}\right|_{r=a}$$

From

$$(\underset{\epsilon_0 \vec{E}_2}{\underset{\uparrow}{\vec{D}_2}} - \underset{\epsilon \vec{E}_1}{\underset{\uparrow}{\vec{D}_1}}) \cdot \vec{n}_{21} = \overset{\sigma=0}{\overset{\downarrow}{0}} \implies \left. -\epsilon \frac{\partial \phi_{in}}{\partial r}\right|_{r=a} = \left. -\epsilon_0 \frac{\partial \phi_{out}}{\partial r}\right|_{r=a}$$

$$-\frac{1}{a}\sum_{l} A_l\, a^{l} \frac{\partial}{\partial\theta} P_l(\cos\theta) = -\frac{1}{a}\left[-E_0\, a \frac{\partial}{\partial\theta} P_1 + \sum_{l} C_l\, a^{-(l+1)} \frac{\partial}{\partial\theta} P_l\right]$$

- No need to calculate $\frac{\partial}{\partial\theta} P_l(\cos\theta)$
  Just match coefficients

- $l=1$ and $l\neq 1$ are different

	BC1	BC2
$\ell=1$	$A_1 = -E_0 + \frac{C_1}{a^3}$	$\frac{\epsilon}{\epsilon_0} A_1 = -E_0 - 2\frac{C_1}{a^3}$
$\ell\neq 1$	$A_\ell = \frac{C_\ell}{a^{2\ell+1}}$	$\frac{\epsilon}{\epsilon_0} \ell A_\ell = -\frac{C_\ell}{a^{2\ell+1}}(\ell+1)$

Solutions: $A_\ell = C_\ell = 0, \ \ell \neq 1$

$$A_1 = -\left(\frac{3}{2+\frac{\epsilon}{\epsilon_0}}\right) E_0 \quad ; \quad C_1 = \left(\frac{\frac{\epsilon}{\epsilon_0}-1}{\frac{\epsilon}{\epsilon_0}+2}\right) a^3 E_0$$

$$\phi_{in} = -\left(\frac{3}{2+\frac{\epsilon}{\epsilon_0}}\right) E_0 \underbrace{r\cos\theta}_{z} \quad ; \quad \phi_{out} = -E_0 \underbrace{r\cos\theta}_{z} +$$

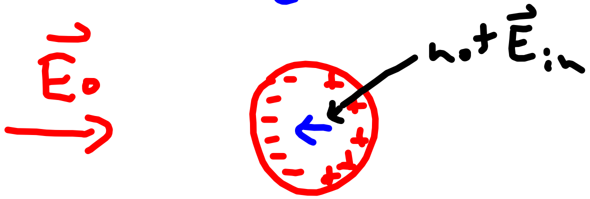
$$+\left(\frac{\frac{\epsilon}{\epsilon_0}-1}{\frac{\epsilon}{\epsilon_0}+2}\right) E_0 \frac{a^3}{r^2}\cos\theta$$

$$\vec{E} = -\nabla \phi$$

$$|\vec{E}_{inside}| = \frac{3}{2 + \frac{\epsilon}{\epsilon_0}} |\vec{E}_0| < |\vec{E}_0|$$

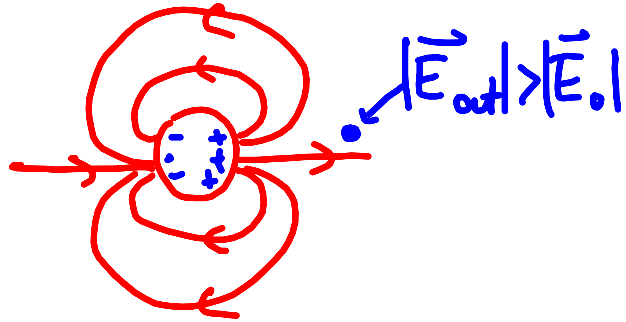
$\frac{\epsilon}{\epsilon_0} > 1$

$\xrightarrow[\epsilon \to \infty]{} 0$

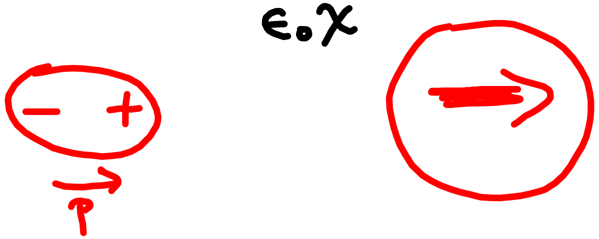


Internal field opposite to $\vec{E}_0$

$$\vec{E} = \vec{E}_0 + \vec{E}_{dipole}$$



$$\vec{P} = \underbrace{(\epsilon - \epsilon_0)}_{\epsilon_0 \chi} \vec{E} = 3\epsilon_0 \left(\frac{\frac{\epsilon}{\epsilon_0} - 1}{\frac{\epsilon}{\epsilon_0} + 2} \right) \vec{E}_0$$

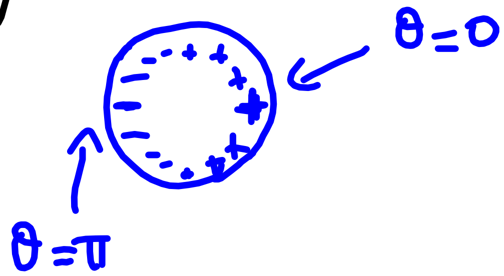


$$\sigma_{pol} = 3\epsilon_0 \left(\frac{\frac{\epsilon}{\epsilon_0} - 1}{\frac{\epsilon}{\epsilon_0} + 2} \right) E_0 \cos\theta$$

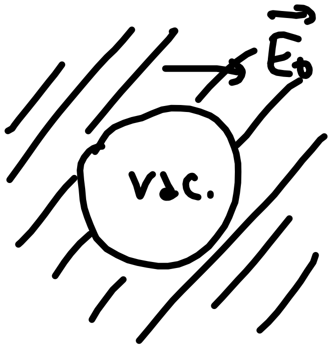
$\Delta \vec{P}$

$\vec{P}_{in}$ ✓

$\vec{P}_{out} = 0$

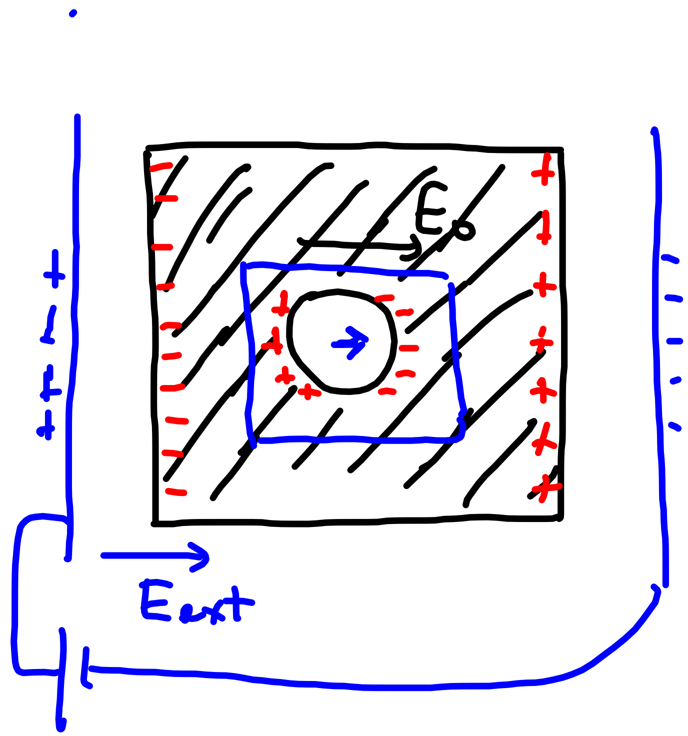


Switch $\epsilon \longleftrightarrow \epsilon_0$



$$E_{in} = \frac{3E_0}{2 + \frac{\epsilon_0}{\epsilon}} > E_0$$

$\frac{\epsilon_0}{\epsilon} \rightarrow < 1$


E_0
E_{ext}

Ch. 5 Magnetostatics

5.1 Introduction

Relations between currents and magnetic fields.

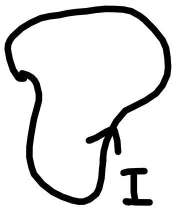
$$\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0 \quad , \quad \text{continuity equation}$$

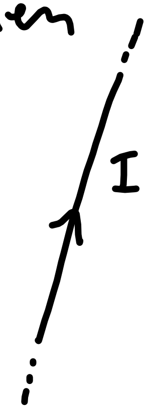
current density

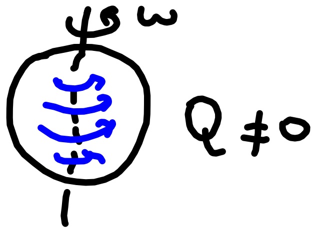
Time-indep. in Ch. 5, then

$$\boxed{\nabla\cdot\vec{J} = 0}$$

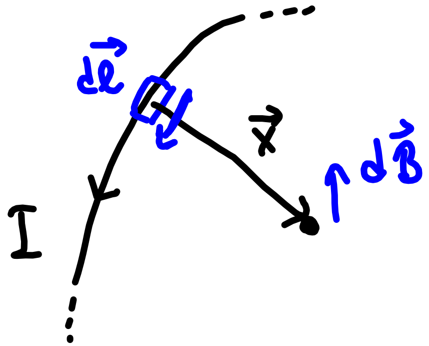
Steady state





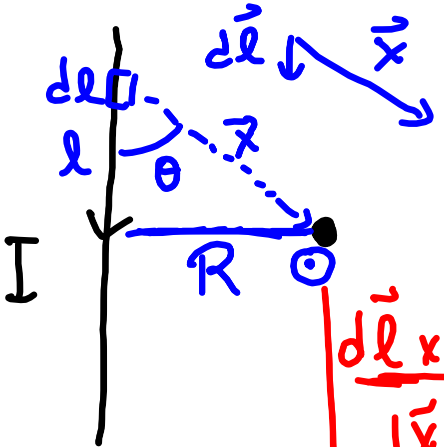


5.2 Biot-Savart law



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{(d\vec{\ell} \times \vec{x})}{|\vec{x}|^3}$$

permeability of space



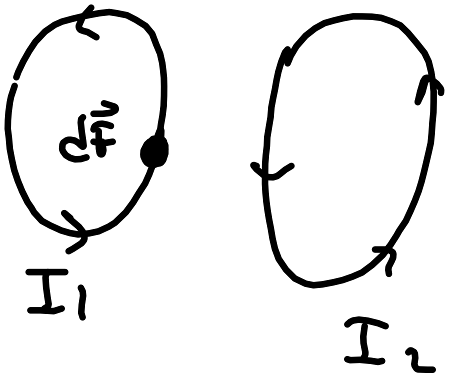
$$|\vec{B}| = \frac{\mu_0}{4\pi} I \int_{-\infty}^{+\infty} d\ell \frac{R}{(\ell^2+R^2)^{3/2}} = \frac{\mu_0 I}{2\pi R}$$

$$|\vec{x}| = \sqrt{\ell^2+R^2}$$

$$\left|\frac{d\vec{\ell} \times \vec{x}}{|\vec{x}|^3}\right| = d\ell \frac{\overbrace{\cancel{\sqrt{\ell^2+R^2}}}^{|\vec{x}|}}{\underbrace{(\ell^2+R^2)^{3/2}}_{|\vec{x}|^3}} \overbrace{\frac{R}{\cancel{\sqrt{\ell^2+R^2}}}}^{\sin\theta}$$

$\vec{F} = q(\cancel{\vec{E}} + \vec{v} \times \vec{B})$, $d\vec{F} = \overbrace{\frac{dq}{dt}}^{I} (d\vec{\ell} \times \vec{B})$

$\vec{v} = \frac{d\vec{\ell}}{dt}$



$\underset{\substack{\text{on 1} \\ \text{caused} \\ \text{by 2}}}{\vec{F}} = \oint_1 I_1 (d\vec{\ell}_1 \times \underset{\substack{\text{caused} \\ \text{by 2}}}{\vec{B}})$

use B.S. law