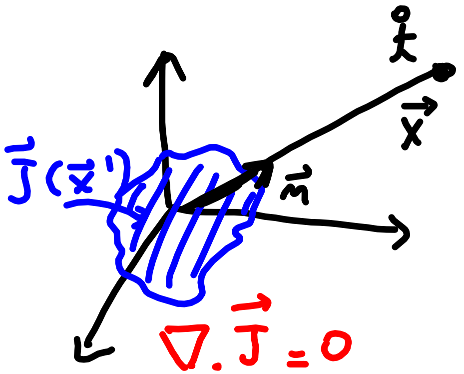
5.6 Magnetic fields of a localized steady current distribution



$$\nabla \cdot \vec{J} = 0$$

$$\frac{1}{|\vec{x}-\vec{x}\,'|} = \frac{1}{|\vec{x}|\left|\hat{n} - \frac{\vec{x}\,'}{|\vec{x}|}\right|} = \frac{1}{|\vec{x}|\sqrt{\left(\hat{n}-\frac{\vec{x}\,'}{|\vec{x}|}\right)\cdot\left(\hat{n}-\frac{\vec{x}\,'}{|\vec{x}|}\right)}}$$

$$\vec{x} = |\vec{x}|\,\hat{n}$$

$\frac{1}{|\vec{x}|}$ "small"

$$\approx \frac{1}{|\vec{x}|}\left(1 + \frac{\vec{x}\cdot\vec{x}\,'}{|\vec{x}|^2}\right)$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{x}\,')\,d^3x'}{|\vec{x}-\vec{x}\,'|} \approx \frac{\mu_0}{4\pi}\int \vec{J}(\vec{x}\,')\left(\frac{1}{|\vec{x}|} + \frac{\vec{x}\cdot\vec{x}\,'}{|\vec{x}|^3}\right)d^3x'$$

$$\int \vec{J}(\vec{x}\,')\,d^3x' = 0$$

$$\vec{A}(\vec{x}) \approx \frac{\mu_0}{4\pi}\int \vec{J}(\vec{x}')\frac{(\vec{x}\cdot\vec{x}')}{|\vec{x}|^3}d^3x' \overset{HW}{=} \frac{\mu_0}{4\pi}\frac{\boxed{\left[\frac{1}{2}\int[\vec{x}'\times\vec{J}(\vec{x}')]d^3x'\right]}\times\vec{x}}{|\vec{x}|^3}$$

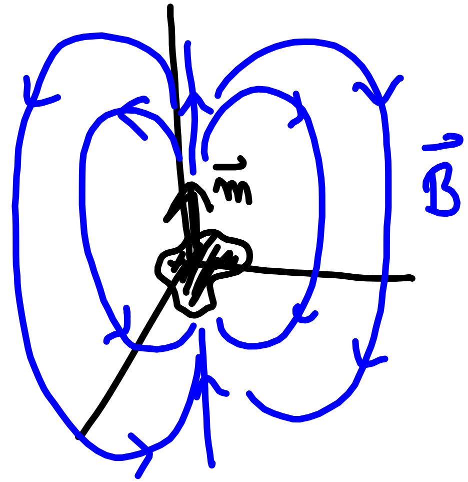
$\vec{m}$ = magnetic moment

$$= \frac{\mu_0}{4\pi}\left(\frac{\vec{m}\times\vec{x}}{|\vec{x}|^3}\right)$$

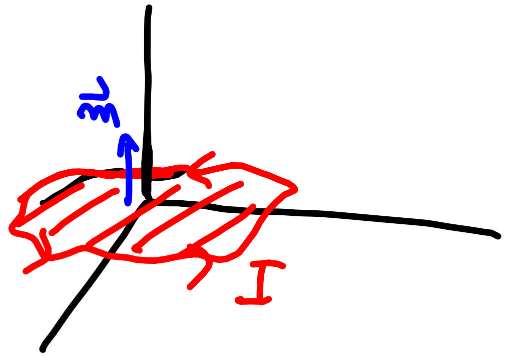
$$\vec{B} = \nabla\times\vec{A} \approx \nabla\times\left[\frac{\mu_0}{4\pi}\frac{(\vec{m}\times\vec{x})}{|\vec{x}|^3}\right] \overset{HW?}{=} \frac{\mu_0}{4\pi}\frac{(3\vec{n}(\vec{n}\cdot\vec{m})-\vec{m})}{|\vec{x}|^3}$$

$$\vec{n} = \frac{\vec{x}}{|\vec{x}|}$$

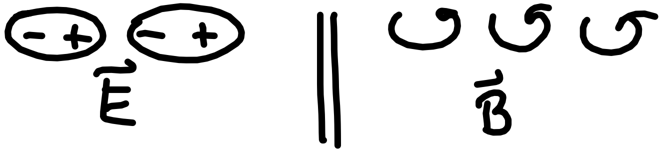
$\vec{m} = m\,\hat{e}_z$



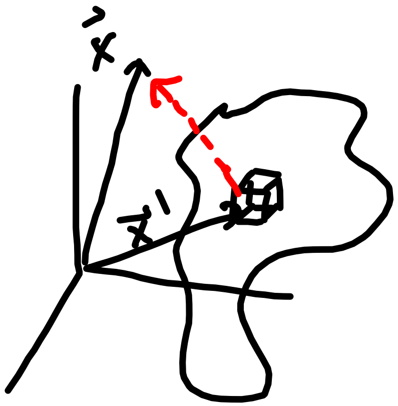
Like a dipole



$\vec{m} = I \cdot \left(\begin{array}{c}\text{Area} \\ \text{of loop}\end{array}\right)$



5.8 Macroscopic equations ; B. Conditions



$\vec{x} \to \vec{x} - \vec{x}'$.

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} + \vec{M}(\vec{x}') \times \underbrace{\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}}_{\nabla'\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)}\right] d^3x'$$

HW!

$$= \frac{\mu_0}{4\pi}\int \frac{\overbrace{\left[\vec{J}(\vec{x}') + \nabla' \times \vec{M}(\vec{x}')\right]}^{\vec{J}_{eff}(\vec{x}')}}{|\vec{x}-\vec{x}'|} d^3x'$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}_{eff} = \mu_0\left[\vec{J} + \nabla \times \vec{M}\right]}$$

$$\vec{H} \underset{\text{def}}{=} \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

; $\vec{H} \underset{\text{def}}{=} \frac{\vec{B}}{\mu_0} - \vec{M}$

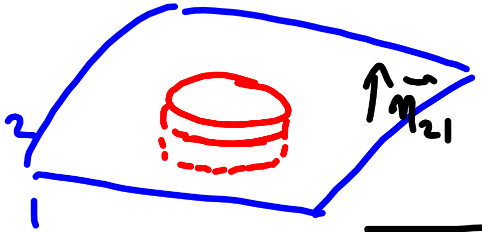
B = magnetic induction | magnetic field

H = magnetic field | H

$\vec{H} = \vec{H}(\vec{B})$
→ nonlinear
→ permanent magnet $\vec{M} = \text{constant}$
→ linear $\vec{B} = \mu \vec{H}$

magnetic permeability

Boundary conditions



$$\int \nabla\cdot\vec B\, d^3x' = 0 = \oint_S \vec B\cdot\vec n\, da$$

$\nabla\cdot\vec B = 0$

$$\boxed{(\vec B_2 - \vec B_1)\cdot\vec n_{21} = 0}$$

$$\boxed{\vec n_{21}\times(\vec H_2 - \vec H_1) = \vec K}$$

surface current

(analog of σ) ch.4

$$\boxed{\begin{array}{c}\nabla\times\vec H = \vec J \\ \nabla\cdot\vec B = 0\end{array}}$$

$$\boxed{\vec H \underset{\text{def}}{=} \frac{\vec B}{\mu_0} - \vec M} \;;\; \boxed{\begin{array}{c}\text{if linear} \\ \vec B = \mu\vec H\end{array}}$$

5.9 How to solve problems?

(a) $\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A}$

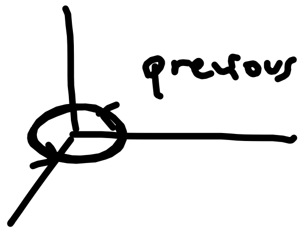
Linear $\vec{B} = \mu \vec{H}$

$\nabla \times \vec{H} = \vec{J} \longrightarrow \nabla \times \frac{\vec{B}}{\mu} = \frac{1}{\mu} \nabla \times (\nabla \times \vec{A}) = \frac{1}{\mu} \left[\nabla \underbrace{(\nabla \cdot \vec{A})}_{=0} - \nabla^2 \vec{A} \right]$

Coul. gauge

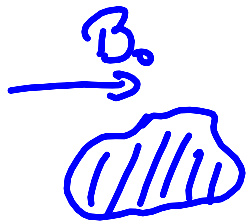
$$\boxed{\nabla^2 \vec{A} = -\mu \vec{J}}$$

Linear material
Coulomb gauge



previous lecture

(B) If $\vec{J}=0 \Rightarrow \nabla\times\vec{H}=0$



$$\vec{H}=-\nabla\Phi_M$$

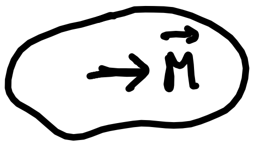
scalar magnetic field

$$\boxed{\nabla^2\Phi_M=0}$$

Laplace Eq.

(C) Assume $\vec{J}=0 \rightarrow \nabla\times\vec{H}=0$

Hard ferromagnet

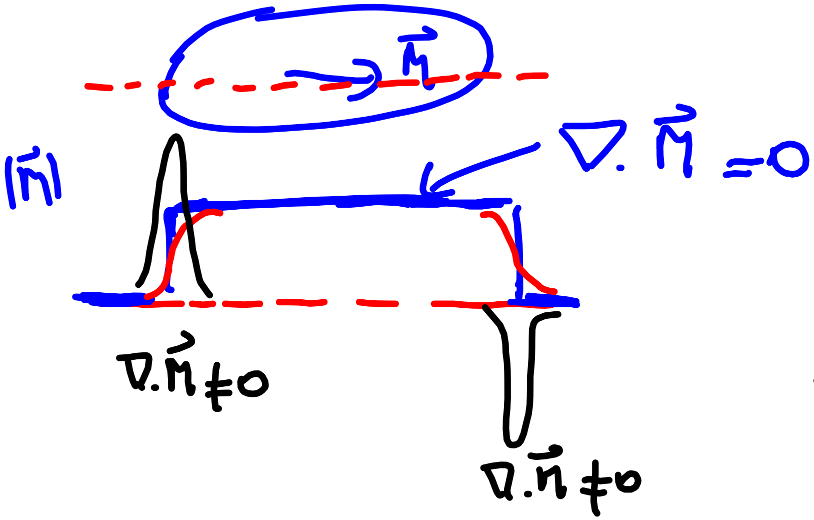


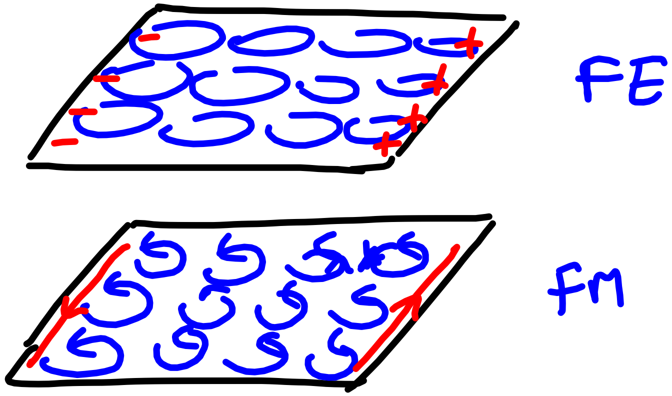
$$\vec{H}=-\nabla\phi_M$$

$$\nabla\cdot\vec{B}=0=\nabla\cdot\left[\mu_0(\vec{H}+\vec{M})\right]$$

$$\boxed{\nabla^2\Phi_M=\nabla\cdot\vec{M}}$$

source





$$\Phi_E = \frac{1}{4\pi}\oint_S \frac{\overbrace{\sigma_E}^{\hat{n}\cdot\vec{P}}\, ds}{|\vec{x}-\vec{x}'|} \quad ; \quad \Phi_M = \frac{1}{4\pi}\oint_S \frac{\overbrace{\sigma_M}^{\hat{n}\cdot\vec{M}}\, ds}{|\vec{x}-\vec{x}'|}$$