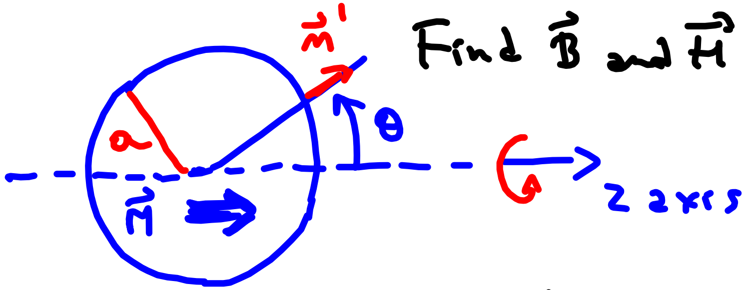
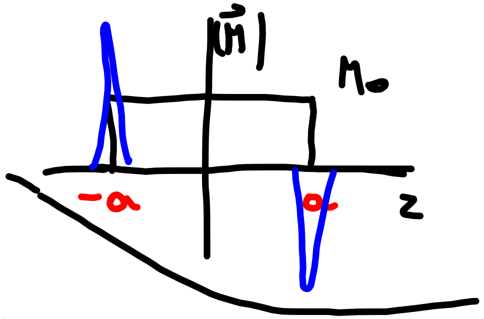
5.10 Uniformly magnetized sphere



Find $\vec B$ and $\vec H$

$\vec M$ = constant

$\vec M = M_0\,\hat e_z$



Since $\vec J = 0 \longrightarrow \Phi_M$

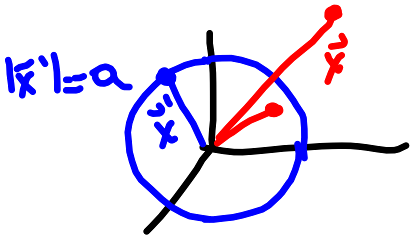
$\nabla \times \vec H = \cancel{\vec J}\;0$

$\vec H = -\nabla \Phi_M$

$$\Phi_M = \frac{1}{4\pi}\oint_S \frac{\overbrace{\vec n'\cdot\vec M(\vec x')}^{=\sigma}}{|\vec x-\vec x'|}\,dS' = \frac{1}{4\pi}\int \frac{M_0\cos\theta'\,a^2\,d\Omega'}{|\vec x-\vec x'|}$$

source of Φ_M is $\nabla\cdot\vec M$

$$\frac{1}{|\vec x-\vec x'|} = 4\pi\sum_{l,m}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}\,Y^*_{lm}(\theta',\phi')\,Y_{lm}(\theta,\phi)$$

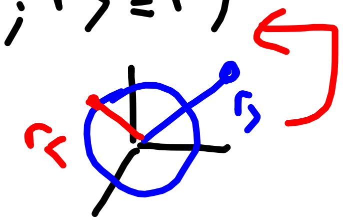


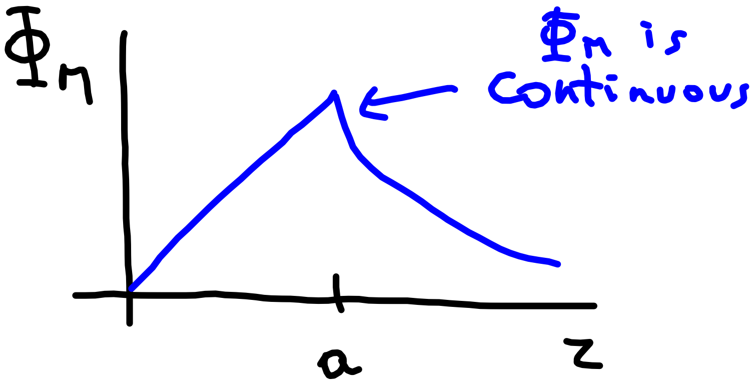
$r_<$ = smallest of r and $\cancel{r'}$ a

$r_>$ = largest of r and $\cancel{r'}$ a

$$\cos\theta' = \sqrt{\frac{4\pi}{3}}\, Y_{10}(\theta',\varphi') \; ; \qquad \int d\Omega'\, Y^*_{lm}(\theta',\varphi')\, Y_{10}(\theta',\varphi') = \delta_{l1}\delta_{m0}$$

$$\Phi_M = \begin{cases} \frac{M_0}{3} r\cos\theta = \frac{M_0}{3} z & \text{inside } (r_< = r;\ r_> = a) \\ \frac{M_0}{3}\left(\frac{a}{r}\right)^3 \underset{\uparrow\, r\cos\theta}{z} & \text{outside } (r_< = a;\ r_> = r) \end{cases}$$



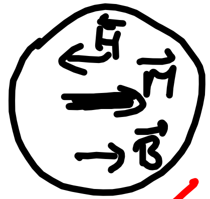


Inside

$$\vec H = -\nabla \Phi_M \,, \quad \Phi_M = \frac{M_0}{3} z$$

$$\vec H = \boxed{-\frac{M_0}{3}\hat e_z}$$

$$\vec B \underset{\text{def of } \vec H}{=} \mu_0(\vec H + \vec M) = \boxed{\frac{2}{3}\mu_0 M_0 \hat e_z}$$



$\cancel{\vec B = \mu \vec H}$

Outside

$$\Phi_M = \frac{M_0}{3}\left(\frac{a}{r}\right)^3 z$$

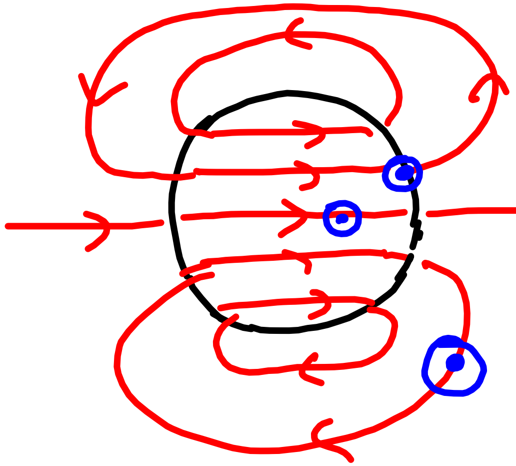
$$\vec M = 0 \; ; \quad \vec B = \mu_0 \vec H = \boxed{\frac{2}{3}\mu_0 M_0 \left(\frac{a}{z}\right)^3 \hat e_z}$$

$$\vec H = -\nabla \Phi_M = \boxed{\frac{2}{3} M_0 a^3 \frac{1}{z^3} \hat e_z}$$

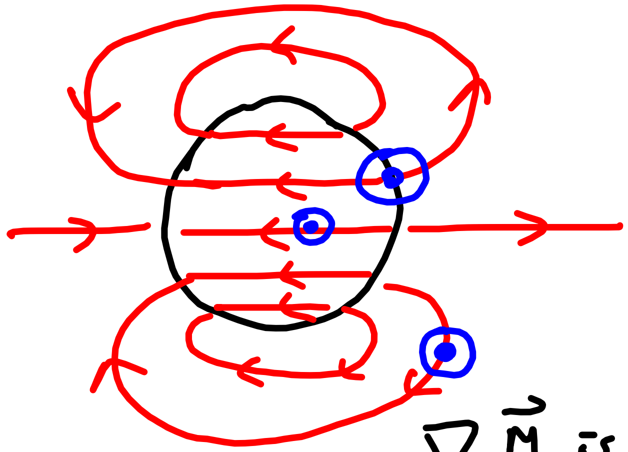
along the z axis

$\vec B$ is continuous at surface
$\nabla \cdot \vec B = 0$

; $\vec H$ is discontinuous
$\nabla \cdot \vec M$ acts as a source



$\underline{\underline{\nabla \cdot \vec{B} = 0}}$

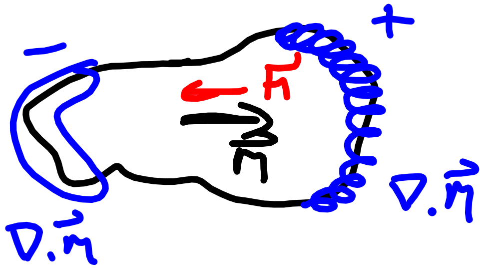


$\nabla \cdot \vec{M}$ is a source

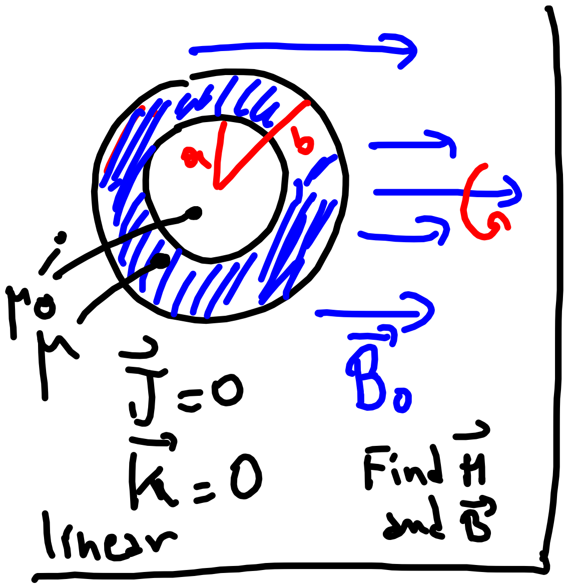
$\vec{B}$ continuous

$\vec{H}$ can be discontinuous

Φ_M is continuous



5.12 Magnetic Shielding



Since $\vec{J}=0 \longrightarrow \vec{H} = -\nabla \Phi_M$

$\vec{B} = \mu \vec{H}$

$\nabla \cdot \vec{B} = 0 = \nabla \cdot (\mu \vec{H}) = \mu \nabla \cdot \vec{H} = 0$

$$\boxed{\nabla^2 \Phi_M = 0}$$

Laplace Eq.

$$\Phi_{M}\Big|_{r>b} = -H_0 \frac{r\overbrace{\cos\theta}^{P_1(\cos\theta)}}{2} + \sum_{l=0}^{\infty} \frac{\alpha_l}{r^{l+1}} P_l(\cos\theta) \quad \text{unknowns}$$

$$\Phi_{M}\Big|_{a<r<b} = \sum_{l=0}^{\infty} \left(\beta_l r^l + \frac{\gamma_l}{r^{l+1}} \right) P_l(\cos\theta)$$

$$\Phi\Big|_{r<a} = \sum_{l=0}^{\infty} \delta_l r^l P_l(\cos\theta)$$

Boundary conditions:

$$(\vec{B}_2 - \vec{B}_1)\cdot \vec{n}_{21} = 0$$

$$\vec{n}_{21} \times (\vec{H}_2 - \vec{H}_1) = 0$$

$$\underbrace{\vec{M}_{21}}_{\hat{e}_r} \times \underbrace{(\vec{H}_2 - \vec{H}_1)}_{\Delta\vec{H}} = 0$$

$$\begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_\phi \\ 1 & 0 & 0 \\ \Delta H_r & \Delta H_\theta & 0 \end{vmatrix} = 0 = \underline{\hat{e}_\phi \, \Delta H_\theta}$$

$$\hookrightarrow H_2\Big|_\theta = H_1\Big|_\theta \quad ; \quad \nabla = \cdots + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{e}_\theta + \cdots$$

$$\boxed{\underbrace{-\frac{1}{b}\frac{\partial\Phi_M}{\partial\theta}\Big|_b}_{r>b} = \underbrace{-\frac{1}{b}\frac{\partial\Phi_M}{\partial\theta}\Big|_b}_{a<r<b} \; ; \; \underbrace{-\frac{1}{a}\frac{\partial\Phi_M}{\partial\theta}\Big|_a}_{r<a} = \underbrace{-\frac{1}{a}\frac{\partial\Phi_M}{\partial\theta}\Big|_a}_{a<r<b}}$$

two equations

$(\vec{B}_2 - \vec{B}_1)\cdot \underbrace{\vec{n}_{21}}_{\hat{e}_r} = 0$

$\mu \vec{H}$

$\nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}, \sim\right)$

$$\boxed{\mu_0 \frac{\partial \Phi}{\partial r}\Big|_b^{r>b} = \mu \frac{\partial \Phi}{\partial r}\Big|_b^{a<r<b} \; ; \; \mu_0 \frac{\partial \Phi}{\partial r}\Big|_a^{r<a} = \mu \frac{\partial \Phi}{\partial r}\Big|_a^{a<r<b}}$$

For $l \neq 1$, all coeff. cancel

For $l = 1$, they are nonzero

$\alpha_1, \beta_1, \gamma_1, \delta_1 \neq 0$

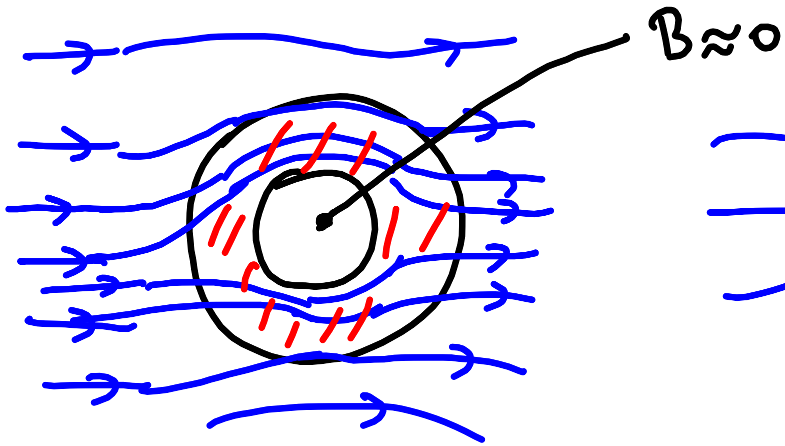
We have 4 equations

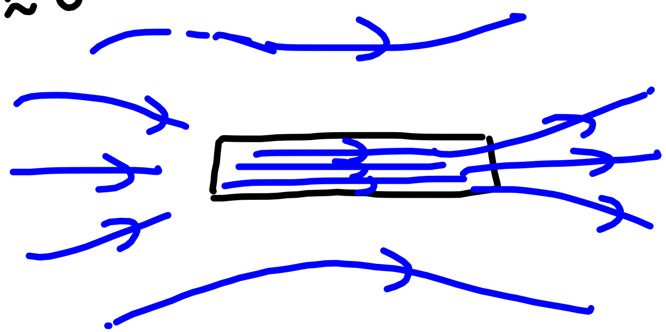
$\delta_1 \sim \frac{1}{(\mu/\mu_0)}$ at large μ

$\frac{\mu}{\mu_0} \sim 100{,}000$

Mu-metals

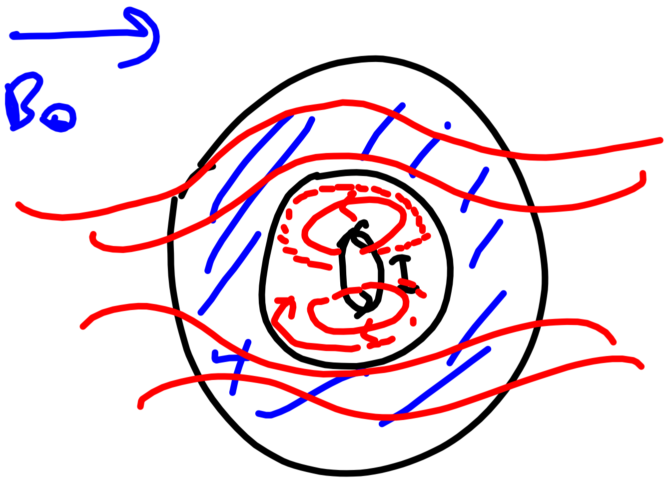
μ-metals

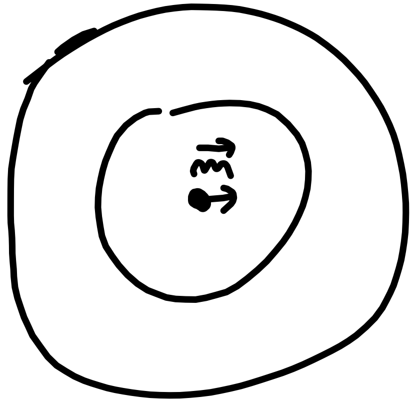


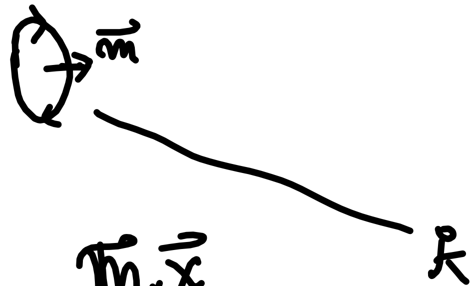


μ-metal, Wikipedia $\frac{\mu}{\mu_0} \gg\gg 1$

50% lectures







$$\frac{\vec{m}\cdot\vec{x}}{|\vec{x}|^3}$$