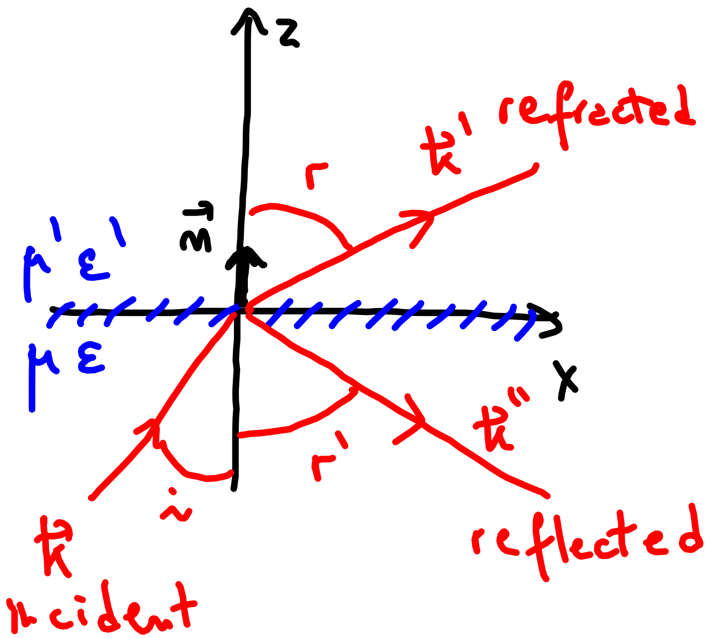
7.3 Reflection and Refraction of waves

We use plane waves of Sec. 7.1

"plane of incidence"



Incident $\vec{E} = \vec{E}_0 e^{i\vec{k}\cdot\vec{x}} e^{-i\omega t}$

$\vec{B} = \sqrt{\mu\varepsilon}\left(\frac{\vec{k}\times\vec{E}}{k}\right)$; $\frac{\vec{k}}{k} = \hat{n}$

(7.11)

(7.5) $\sqrt{\mu\varepsilon} = \frac{n}{c}$

Refracted $\vec{E}' = \vec{E}_0' e^{i\vec{k}'\cdot\vec{x}} e^{-i\omega t}$

$\vec{B}' = \sqrt{\mu'\varepsilon'}\left(\frac{\vec{k}'\times\vec{E}'}{k'}\right)$

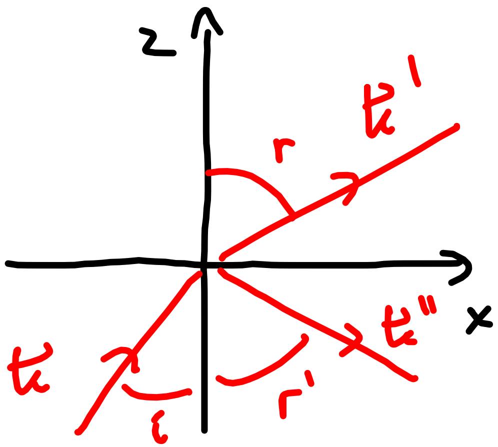
Reflected

$\vec{E}'' = \vec{E}_0'' e^{i\vec{k}''\cdot\vec{x}} e^{-i\omega t}$

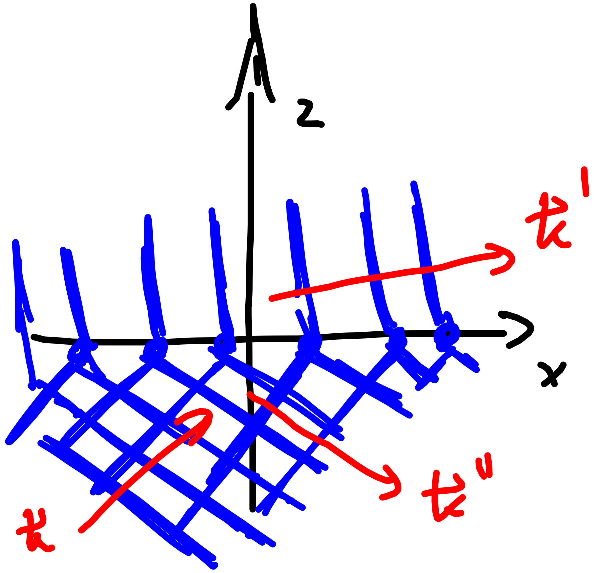
$\vec{B}'' = \sqrt{\mu\varepsilon}\left(\frac{\vec{k}''\times\vec{E}''}{k''}\right)$

(7.4) $k = \sqrt{\mu\varepsilon}\,\omega$

$k'' = k$



$\equiv$



The "peaks" have to match at z=0

$$\underbrace{\vec{k}\cdot\vec{x}\Big|_{z=0}}_{\cancel{k_z z}+k_x x\big|_{z=0}} = \vec{k}'\cdot\vec{x}\Big|_{z=0} = \vec{k}''\cdot\vec{x}\Big|_{z=0}$$

$$k_x \cancel{x} = k'_x \cancel{x} = k''_x \cancel{x}$$

$$k\sin(i) = k'\sin(r) = k''\sin(r')$$

k

$$k\sin(i) = k'\sin(r)$$

$$\boxed{\frac{\sin(i)}{\sin(r)} = \frac{k'}{k} = \frac{\sqrt{\mu'\epsilon'}}{\sqrt{\mu\epsilon}} = \frac{n'}{n}}$$

(7.4)

$$\boxed{i = r'}$$

Now we must find $\vec{E}_0'$, $\vec{E}_0''$

Use boundary conditions: (page 18)

Normal component of $\vec{D}$ and $\vec{B}$ are continuous
Tangential component of $\vec{E}$ and $\vec{H}$ are continuous

Interface $\sigma = 0$, $\vec{K} = 0$, linear $\vec{D} = \epsilon \vec{E}$, $\vec{H} = \vec{B}/\mu$ extras

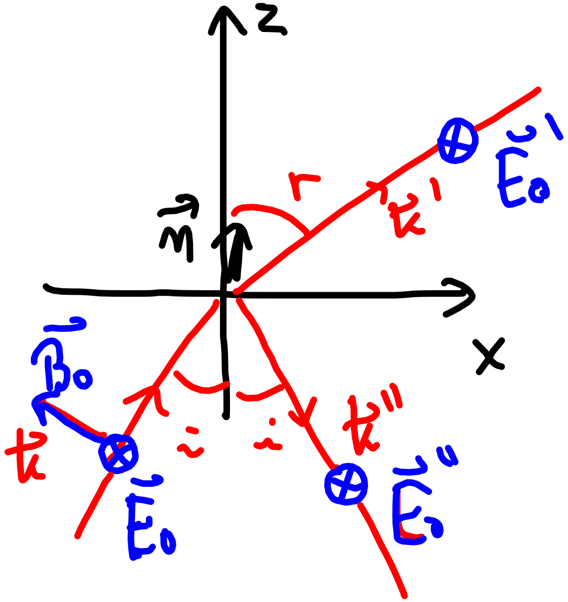
$$(\vec{D}_2 - \vec{D}_1) \cdot \vec{n} = 0$$
$$(\vec{B}_2 - \vec{B}_1) \cdot \vec{n} = 0$$
$$(\vec{E}_2 - \vec{E}_1) \times \vec{n} = 0$$
$$(\vec{H}_2 - \vec{H}_1) \times \vec{n} = 0$$

$$\left[\epsilon(\vec{E}_0 + \vec{E}_0'') - \epsilon' \vec{E}_0'\right] \cdot \vec{n} = 0$$

$k'' = k$

$$(\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'' - \vec{k}' \times \vec{E}_0') \cdot \vec{n} = 0$$
$$(\vec{E}_0 + \vec{E}_0'' - \vec{E}_0') \times \vec{n} = 0$$
$$\left(\frac{\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0''}{\mu} - \frac{\vec{k}' \times \vec{E}_0'}{\mu'}\right) \times \vec{n} = 0$$

Example $\vec{E}_0 \perp$ to "plane of incidence"



First BC Eq gives nothing.

Second BC $\equiv$ Third BC

Third BC
$$\boxed{E_0 + E_0'' - E_0' = 0}$$

Fourth BC provides the second equation

$$(\vec{k} \times \vec{E}_0) \times \vec{n} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ k_x & 0 & k_z \\ 0 & E_0 & 0 \end{vmatrix} \times \vec{n} = \begin{pmatrix} -k_z E_0 \\ 0 \\ k_x E_0 \end{pmatrix} \times \vec{n} =$$

$$= \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ -k_z E_0 & 0 & k_x E_0 \\ 0 & 0 & 1 \end{vmatrix} = k_z E_0 \, \hat{e}_y = k \cos(i) E_0 \, \hat{e}_y \quad \text{incident}$$

$$(\vec{k}' \times \vec{E}_0') \times \vec{n} = k' \cos(r) E_0' \, \hat{e}_y \quad \text{transmitted}$$

$$(\vec{k}'' \times \vec{E}_0'') \times \vec{n} = -k'' \cos(i) E_0'' \, \hat{e}_y \quad \text{reflected}$$

$k'' = k$

$$\frac{k}{\mu} = \omega \sqrt{\frac{\varepsilon}{\mu}}$$

Fourth BC:

$$\boxed{\frac{k \cos(i) E_0}{\mu} - \frac{k \cos(i) E_0''}{\mu} - \frac{k' \cos(r) E_0'}{\mu'} = 0}$$

$$\cos(r) = \sqrt{1 - \sin^2(r)} = \sqrt{1 - \left[\sin(i)\frac{n}{n'}\right]^2}.$$

Snell's law

$$\boxed{\frac{E_0'}{E_0} = \frac{2n\cos(i)}{n\cos(i) + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2(i)}}}$$

$$\boxed{\frac{E_0''}{E_0} = \frac{E_0'}{E_0} - 1}$$

Look in Griffiths

Conservation of energy

$$T + R = 1$$

$$\boxed{\left|\frac{E_0'}{E_0}\right|^2 \sqrt{\frac{\varepsilon'}{\mu'}\frac{\mu}{\varepsilon}} + \left|\frac{E_0''}{E_0}\right|^2 = 1}$$

sum rule

$$u \sim \frac{\epsilon}{2}\vec{E}^2 + \frac{1}{2\mu}\vec{B}^2$$