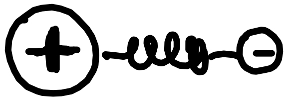
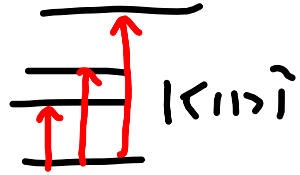
7.5 ϵ is ω dependent



ω_0 characteristic frequency

$\hbar\omega_i$



Add external electric field $\vec{E}(t) = \vec{E}_0 e^{-i\omega t}$

$$\vec{F} = m\ddot{\vec{x}} = -m\omega_0^2\vec{x} - e\vec{E}(t)$$

$$\vec{x} = \vec{x}_0 e^{-i\omega t}$$

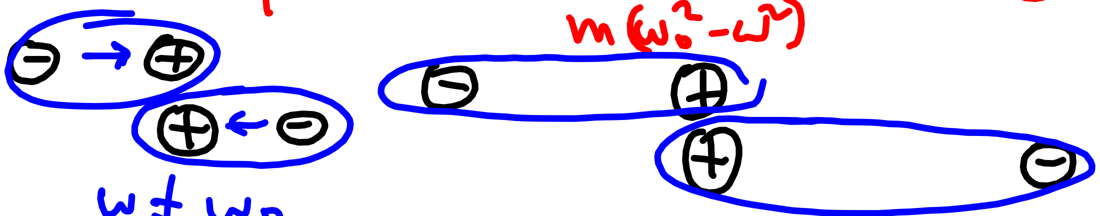
$$\ddot{\vec{x}} = -\omega^2\vec{x}$$

$$-m\omega^2\vec{x}_0\cancel{e^{-i\omega t}} = -m\omega_0^2\vec{x}_0\cancel{e^{-i\omega t}} - e\vec{E}_0\cancel{e^{-i\omega t}}$$

$$|\vec{x}_0| = \frac{e|\vec{E}_0|}{m(\omega_0^2-\omega^2)}$$

$\uparrow$ divergence

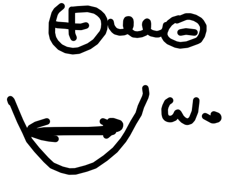
$$|\vec{p}| = e|\vec{x}_0| = \frac{e^2|\vec{E}_0|}{m(\omega_0^2-\omega^2)}$$



$\omega \neq \omega_0$

Add "damping" (coupling to other d.of freedom)

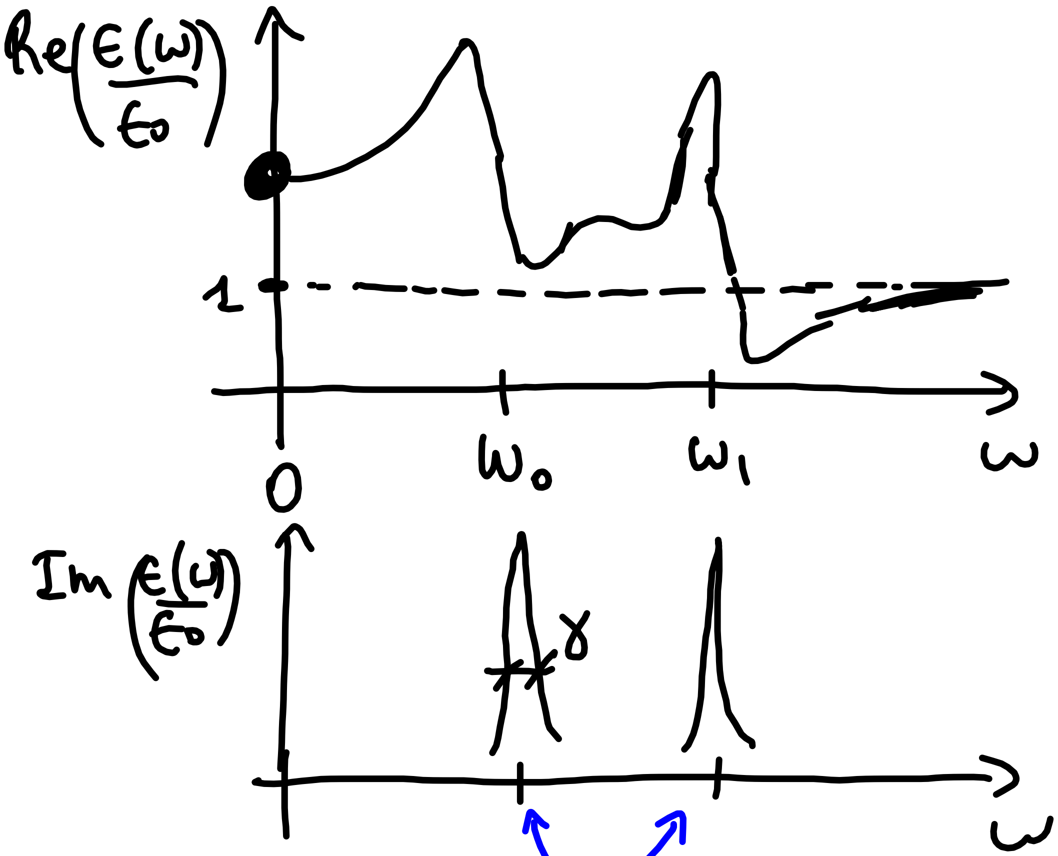
Add term $m\gamma\dot{x}$



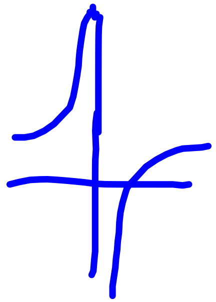
$$\omega_0^2-\omega^2 \longrightarrow \omega_0^2-\omega^2-i\omega\gamma$$

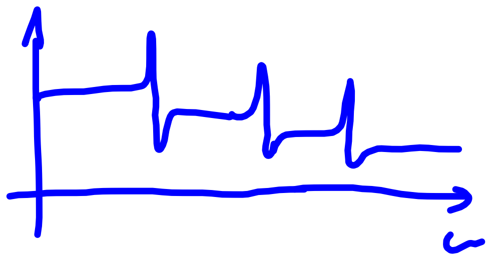
$$\vec{P} = \underbrace{N_{dipoles}\,\frac{e^2\,\vec{E}_0\,e^{-i\omega t}}{m\left(\omega_0^2-\omega^2-i\omega\gamma\right)}}_{\text{microscopic}} = \underbrace{\epsilon_0\,\chi_e\,\vec{E}\;;\;\epsilon=\epsilon_0(1+\chi_e)}_{\text{ch 4 macroscopic}}$$

$$\boxed{\frac{\epsilon(\omega)}{\epsilon_0} = 1+\chi_e = 1+\frac{N_{dipoles}\,e^2}{\epsilon_0 m\left(\omega_0^2-\omega^2-i\omega\gamma\right)}} \longrightarrow 1+\frac{Ne^2}{\epsilon_0 m}\sum_j \frac{f_j}{\left(\omega_j^2-\omega^2-i\omega\gamma_j\right)}$$

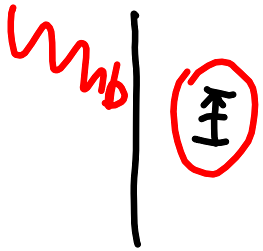


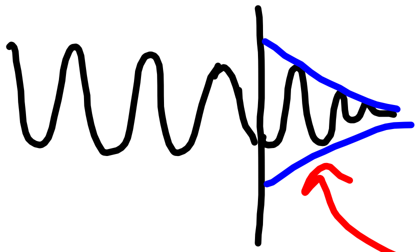
resonant absorption occurs





$\epsilon = \epsilon(\omega)$



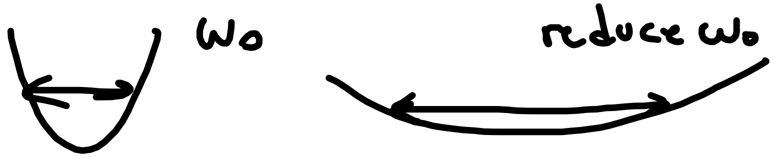


$$e^{ikx} = e^{ik_R x}\, e^{-k_I x}$$

$k = k_R + i k_I$

$$k = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu\,\underbrace{\epsilon(\omega)}_{\epsilon_R + i\epsilon_I}}$$

Low-frequency behavior



$\omega_0 \to 0$

$$\frac{\epsilon(\omega)}{\epsilon_0} = \underbrace{1 + \frac{Ne^2}{\epsilon_0 m}\sum_{j\neq 0}\frac{f_j}{\omega_j^2 - \cancel{\omega^2} - i\cancel{\omega}\gamma_j}}_{\text{constant}} + \frac{Ne^2}{\epsilon_0 m}\cdot\frac{f_0}{(\cancel{\omega_0^2} - \cancel{\omega^2} - i\omega\gamma_0)}$$

$$\epsilon(\omega) = \epsilon_{rest} + \frac{Ne^2 f_0}{m(-i\omega\gamma_0)} = \epsilon_{rest} + \frac{i}{\omega}\frac{Ne^2 f_0}{m\gamma_0} \to \sigma$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \; ; \; \vec{D} = \epsilon_b \vec{E} \; ; \; \vec{J} = \sigma \vec{E} \qquad \text{MACRO}$$

$$\sigma \vec{E} + \epsilon_b \frac{\partial \vec{E}}{\partial t} = (\sigma - i\omega\epsilon_b)\vec{E} = (-i\omega)\underbrace{\left(\epsilon_b + \frac{i\sigma}{\omega}\right)}_{\text{same form as in previous page.}}\vec{E}$$

$$\vec{E} = \vec{E}_0 e^{-i\omega t}$$

$$\frac{\partial \vec{E}}{\partial t} = -i\omega \vec{E}$$

$$\boxed{\begin{aligned} \sigma &= \frac{Ne^2 f_0}{m \gamma_0} \\ \frac{\epsilon_b(\omega)}{\epsilon_0} &= \frac{\epsilon_{rest}}{\epsilon_0} = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_{j \neq 0} \frac{f_j}{\omega_j^2 - \omega^2} \end{aligned}}$$

MACRO
Max. Eqs.

Atomic physics