

1.4 Diff. form of Gauss's law

$$\oint_S (\vec{E} \cdot \vec{n}) da = \frac{1}{\epsilon_0} \int_V \rho(\vec{x}) d^3x = \int_V (\nabla \cdot \vec{E}) d^3x$$

physics

math

$$\nabla \cdot \vec{E} = \frac{\rho(\vec{x})}{\epsilon_0}$$

1.5 Scalar potential

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_V \rho(\vec{x}') \frac{(\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} d^3x'$$

$$= -\nabla_{\vec{x}} \left(\frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{x}') d^3x'}{|\vec{x} - \vec{x}'|} \right)$$

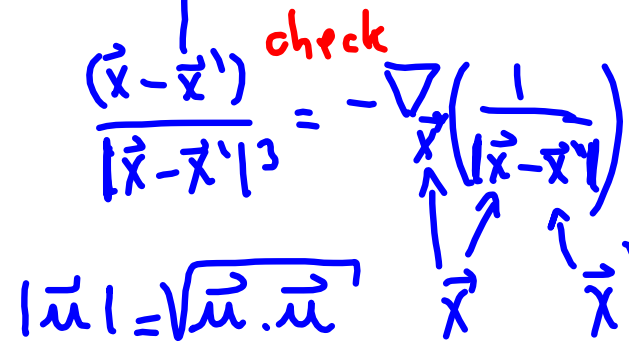
$\Phi(\vec{x})$

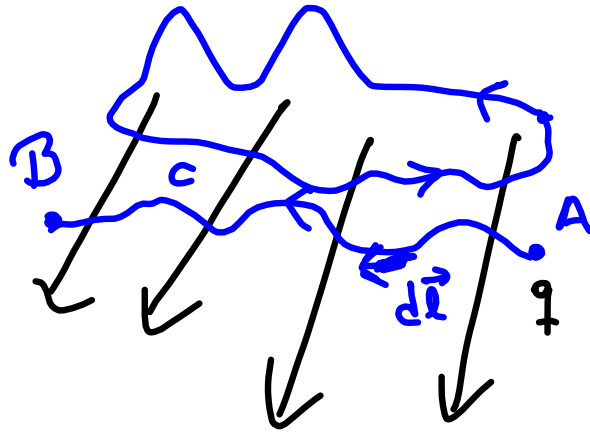
$$\boxed{\vec{E}(\vec{x}) = -\nabla_{\vec{x}} \Phi(\vec{x})}$$

$$\nabla_x \vec{E} = -\nabla_x \nabla \Phi = 0$$

$$\boxed{\nabla_x \vec{E} = 0}$$

check





$$W_{A \rightarrow B} = - \int_A^B \vec{F}_{\text{force}} \cdot d\vec{l} =$$

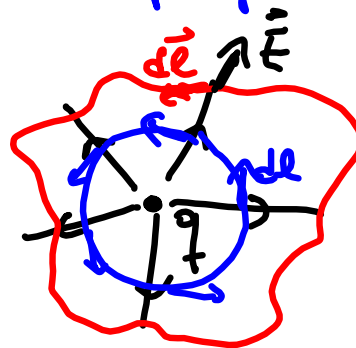
$$= -q \int_A^B \vec{E}(\vec{x}) \cdot d\vec{l} = q \int_A^B \nabla \Phi \cdot d\vec{l} \quad \text{check}$$

$$= q \int_A^B d\Phi = q (\Phi_B - \Phi_A)$$

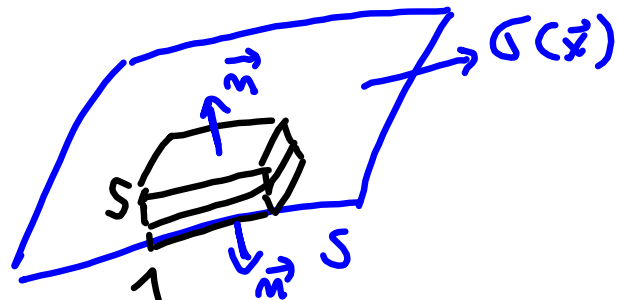
independent of the shape of C

; $q\Phi = \text{potential energy.}$

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

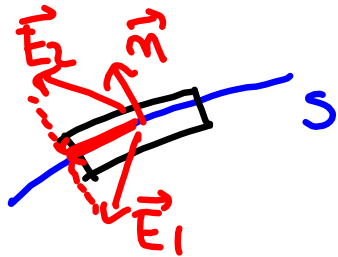
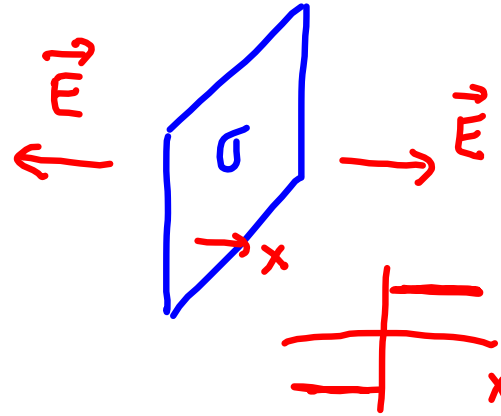


1.6 Surface distribution of charge

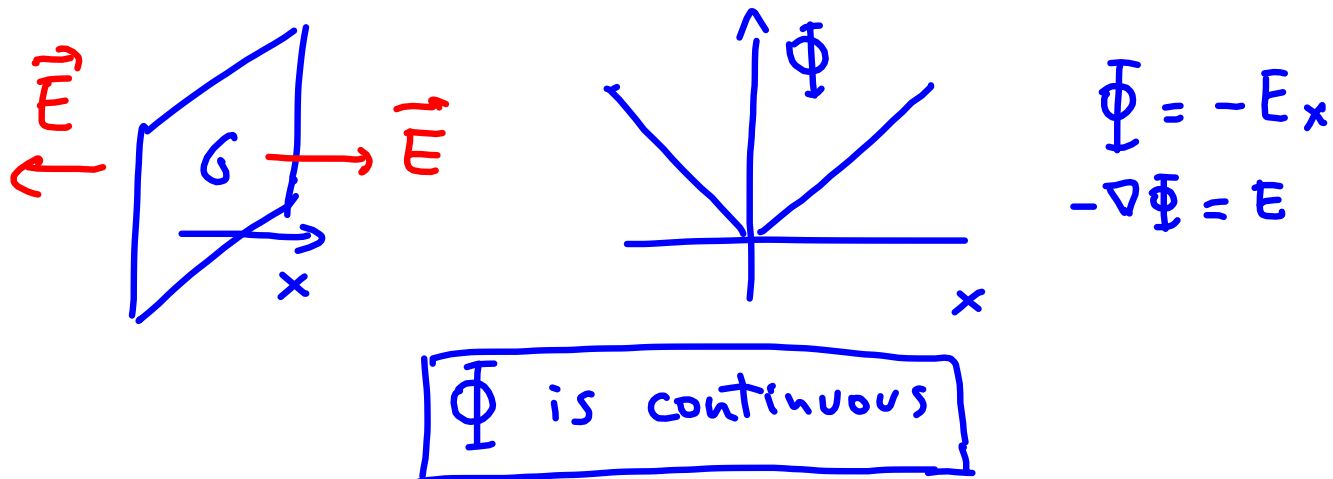


$$\int_S (\vec{E} \cdot \vec{n}) da = \frac{1}{\epsilon_0} \int_S \sigma(\vec{x}) da$$

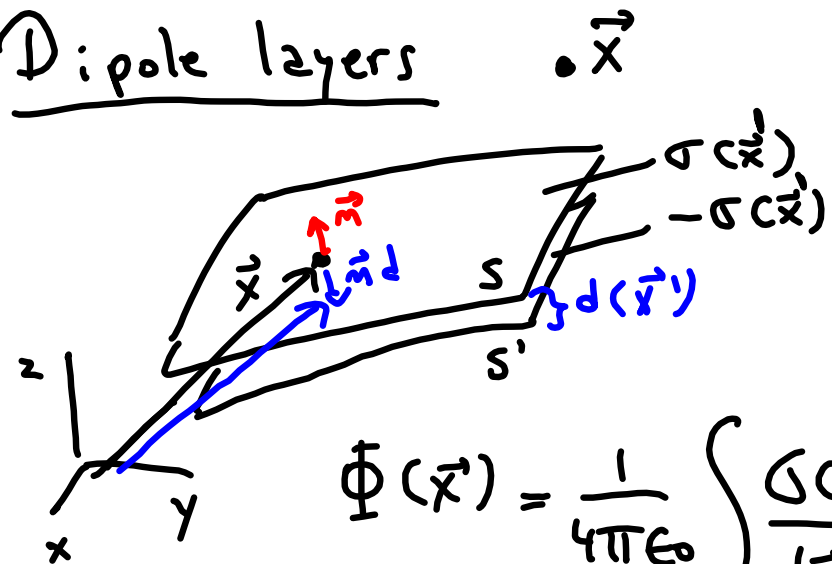
$$(\vec{E}_2 - \vec{E}_1) \cdot \vec{n} = \frac{\sigma(\vec{x})}{\epsilon_0}$$



$\vec{E} \cdot \vec{n}$ discontinuous



Dipole layers



$\Phi(\vec{x})?$



$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma(\vec{x}') da'}{|\vec{x} - \vec{x}'|} - \frac{1}{4\pi\epsilon_0} \int_{S'} \frac{\sigma(\vec{x}') da'}{|\vec{x} - \vec{x}' + \vec{n}'d|}$$

check

$$\frac{1}{|\vec{u} + \vec{\epsilon}|} \approx \frac{1}{|\vec{u}|} + \vec{\epsilon} \cdot \nabla \left(\frac{1}{|\vec{u}|} \right)$$

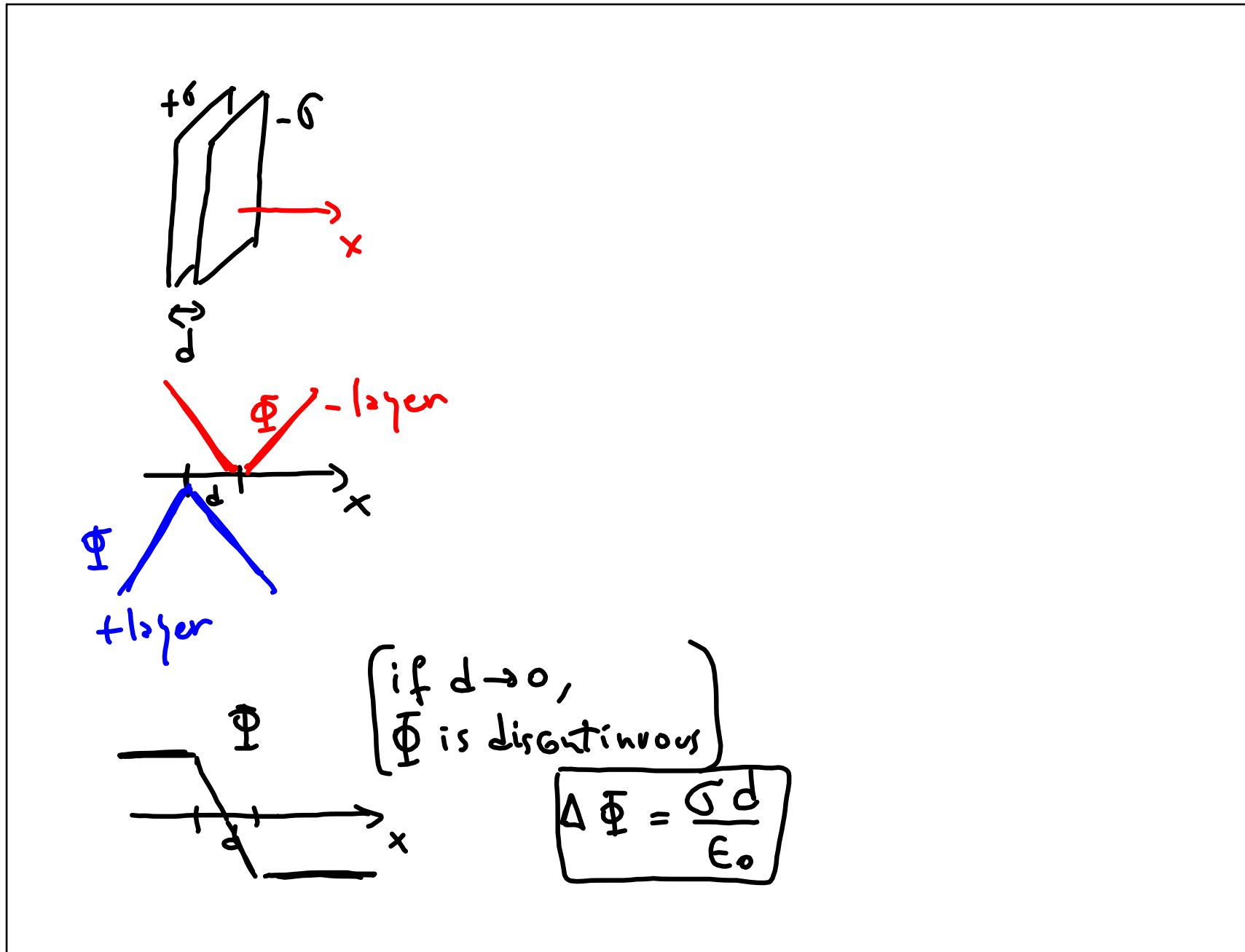
;

$$\nabla_{\vec{x}} = -\nabla_{\vec{x}'}$$

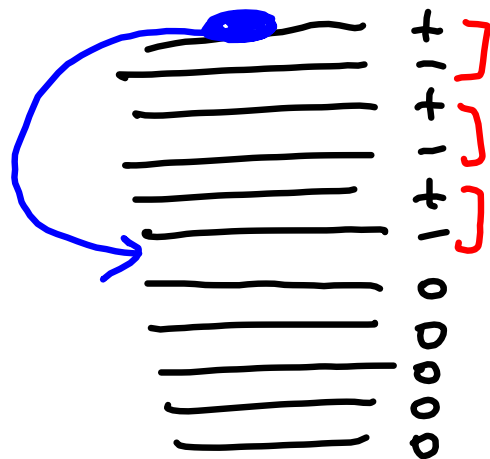
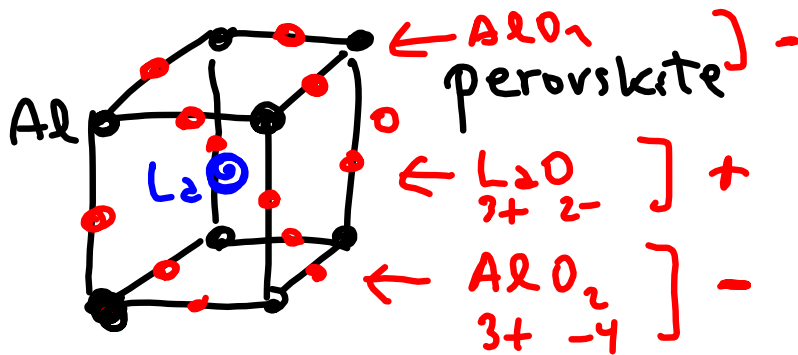
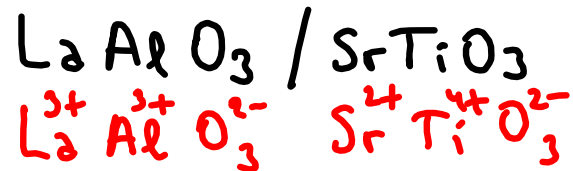
$$\nabla = -\nabla'$$

small

$$\Phi(\vec{x}) \approx \frac{1}{4\pi\epsilon_0} \int_S \sigma(\vec{x}') \vec{n}' d(\vec{x}') \cdot \nabla' \left(\frac{1}{|\vec{x} - \vec{x}'|} \right) da'$$



Oxide interfaces



LaAlO_3

SrTiO_3

"polar catastrophe"

"electronic reconstruction"