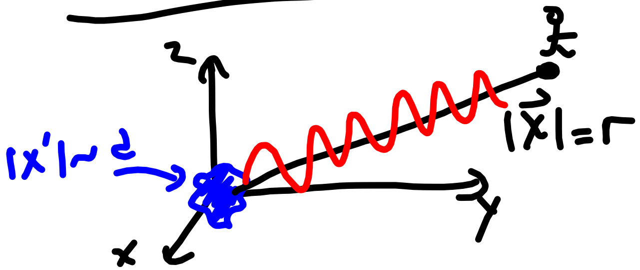
Last lecture



$\lambda = \frac{2\pi c}{\omega}$

$r \ll \lambda$ "near"

$r \gg \lambda$ "far"

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int d^3x' \frac{\vec{J}(\vec{x}')e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} \qquad k=\omega/c$$

"near" $e^{ik|\vec{x}-\vec{x}'|} \sim e^{ir/\lambda} \sim 1 \longrightarrow$ Ch. 1-5 $\vec{A}(\vec{x},t) = \vec{A}(\vec{x})e^{-i\omega t}$

($k \sim 1/\lambda$, $|\vec{x}| \sim r$, $|\vec{x}'| \sim d$)

"far" $\boxed{|\vec{x}-\vec{x}'| = |\vec{x}|\left|\frac{\vec{x}}{|\vec{x}|} - \frac{\vec{x}'}{|\vec{x}|}\right| = r\left|\vec{n} - \frac{\vec{x}'}{r}\right| = r\sqrt{\left(\vec{n}-\frac{\vec{x}'}{r}\right)\cdot\left(\vec{n}-\frac{\vec{x}'}{r}\right)} \approx}$

($\vec{n}\cdot\vec{n} = 1$)

$$\approx r\sqrt{1 - \frac{2\vec{n}\cdot\vec{x}'}{r}} = r\left(1 - \frac{\vec{n}\cdot\vec{x}'}{r}\right) = r - \vec{n}\cdot\vec{x}'$$

$$\left[e^{ik|\vec{x}-\vec{x}'|} \sim e^{ikr}\, e^{-ik\vec{n}\cdot\vec{x}'} \right]$$

$$\vec{A}(\vec{x}) \underset{\text{small } d}{\approx} \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{J}(\vec{x}')\,\underbrace{e^{-ik\vec{n}\cdot\vec{x}'}}_{\text{expand}}\,d^3x'$$

First term

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{J}(\vec{x}')\,d^3x' = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(-i\omega)\underbrace{\int \vec{x}'\rho(\vec{x}')\,d^3x'}_{\vec{p}}$$

$$\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0$$

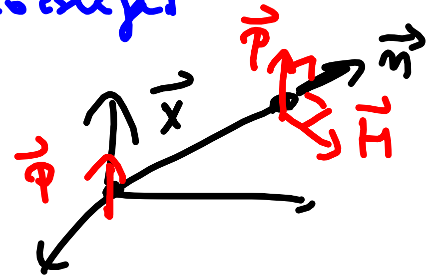
$$\nabla\cdot\vec{J}-i\omega\rho=0$$

localized

$\vec{p}$ electric dipole moment

$$\boxed{\vec{A}(\vec{x}) = -i\omega\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{p}}$$

$$\vec{p}\,e^{-i\omega t}$$



$$\vec{H} = \frac{1}{\mu_0}(\nabla\times\vec{A}) = \frac{-i\omega}{4\pi}\nabla\left(\frac{e^{ikr}}{r}\right)\times\vec{p}$$

$$\nabla\times(\psi\vec{c}) = \nabla\psi\times\vec{c} + \cancel{\psi\nabla\times\vec{c}}$$

$\sim\omega^2$

$$= \frac{ck^2}{4\pi}(\vec{n}\times\vec{p})\,\frac{e^{ikr}}{r}\left(1-\cancel{\frac{1}{ikr}}\right)$$

$\frac{1}{r}$!

Electric field

$$\vec{E} = \frac{i}{k}\sqrt{\frac{\mu_0}{\epsilon_0}}\left[\nabla \times \vec{H}\right] = \text{ugly formula } \underline{9.18}$$

Max. Eq.

↳ Contains both "near" and "far"

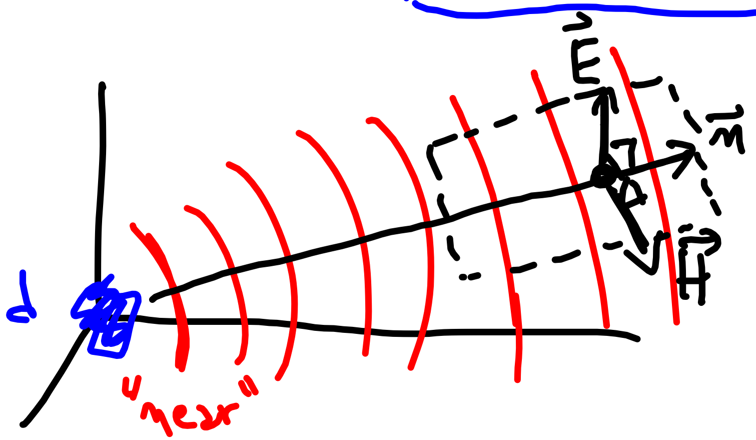
"Far region":

$$\boxed{\vec{E} = \frac{k^2}{4\pi\epsilon_0}(\vec{n}\times\vec{p})\times\vec{n}\,\frac{e^{ikr}}{r}}$$

ω^2

$\frac{1}{r}$ $\left(|\vec{E}| \sim \frac{1}{r^2} \text{ electros.}\right)$

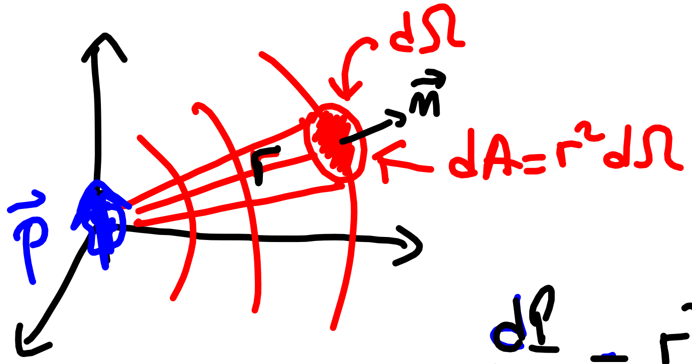
Radiation fields



like plane wave solutions

Power radiated to infinity?

$$\vec{S} = \vec{E} \times \vec{H}$$



$$dP = \underbrace{r^2 d\Omega}_{dA}\ \vec{n} \cdot \vec{S}$$

$$= r^2\, \vec{n} \cdot (\vec{E} \times \vec{H})\, d\Omega$$

$\frac{1}{r}$ $\frac{1}{r}$

$$\frac{dP}{d\Omega} = r^2\, \vec{n} \cdot (\vec{E} \times \vec{H})$$

oscillating

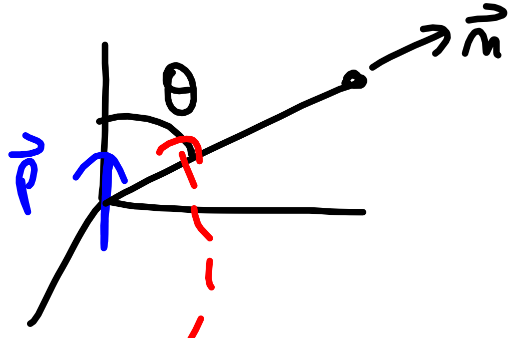
time average

$$\stackrel{\text{time average}}{=} \frac{1}{2} r^2\, \vec{n} \cdot (\vec{E} \times \vec{H}^*)$$

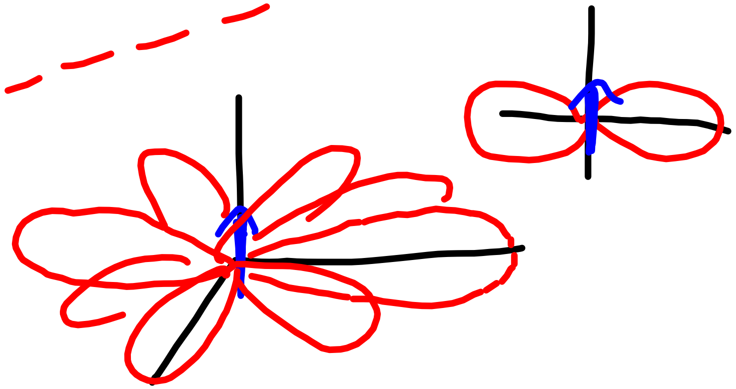
$$\boxed{\frac{dP}{d\Omega} = \frac{1}{2} \frac{c^2 k^4}{(4\pi)^2} \sqrt{\frac{\mu_0}{\epsilon_0}} \left| (\vec{n} \times \vec{p}) \times \vec{n} \right|^2}$$

$$\vec{m} \times \vec{p} = \underbrace{|\vec{m}||\vec{p}|\sin\theta}\ \hat{e}_{\perp \text{ to both } \vec{m} \text{ and } \vec{p}}$$

$$|(\vec{m} \times \vec{p}) \times \vec{m}| = \underbrace{|\vec{m}|}_{1}|\vec{p}|\sin\theta\,\underbrace{|\vec{m}|}_{1}\,\underbrace{\sin(\pi/2)}_{1}$$



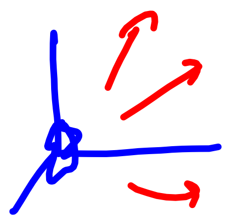
$$\boxed{\frac{dP}{d\Omega} = \frac{c^2 k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\epsilon_0}}\,|\vec{p}|^2 \sin^2\theta}$$

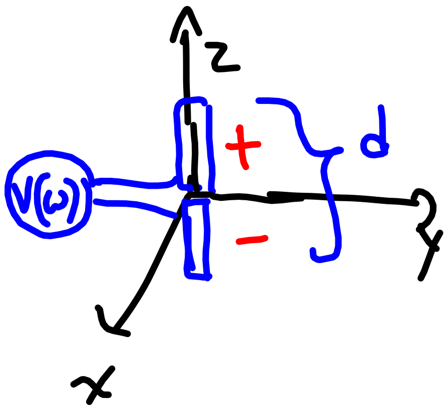


$$\Downarrow$$

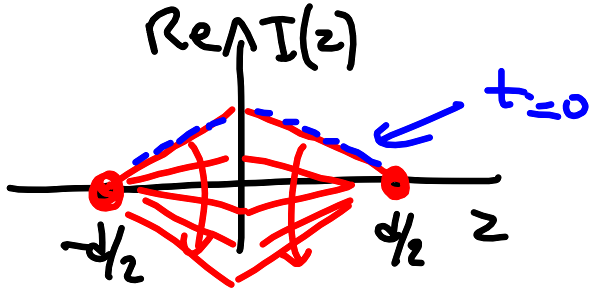
ω^4

$$\boxed{P = \frac{c^2 k^4}{12\pi}\sqrt{\frac{\mu_0}{\epsilon_0}}\,|\vec{p}|^2}$$





$$I(z) = I_0\left(1 - \frac{|z|}{d/2}\right)e^{-i\omega t}$$



$|z| = d/2$

$|\vec{p}|$?

$\vec{p} = \int \vec{x}'\,\rho(\vec{x}')\,d^3x'$

$I(\vec{x}) \longrightarrow \rho(\vec{x})$

$\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$; $\nabla\cdot\vec{J} - i\omega\rho = 0$

$J = I\,\delta(x)\delta(y)$

$$\rho = \frac{\nabla\cdot\vec{J}}{i\omega} = \frac{1}{i\omega}\frac{d}{dz}\left(I_0 - \frac{\cancel{|z|}\ z}{d/2}\right) = \begin{cases} \frac{2iI_0}{\omega d} & z>0 \\ \frac{-2iI_0}{\omega d} & z<0 \end{cases}$$

$$|\vec{p}| = \left|\int \vec{x}'\,\rho(\vec{x}')\,d^3x'\right| = \left|\int_{-d/2}^{+d/2} z'\left(\pm\frac{2iI_0}{\omega d}\right)dz'\right| = \boxed{\frac{I_0 d}{2\omega}}$$

length of wire

$$\boxed{\frac{dP}{d\Omega} = \frac{c^2k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\epsilon_0}}\underbrace{\left(\frac{I_0 d}{2\omega}\right)^2}_{|\vec{p}|^2}\sin^2\theta}$$