9.4 Example of linear antenna

d , λ , r

Before $d \ll \lambda, r$

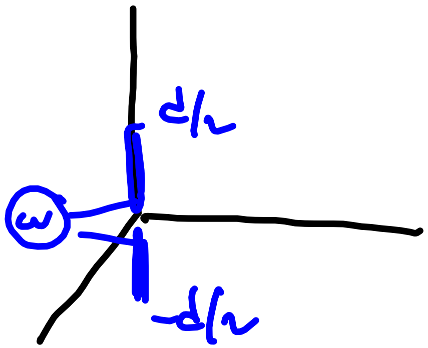
Today $d \sim \lambda \ll r$

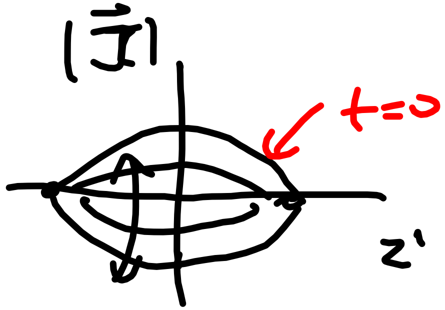
$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int d^3\vec{x}' \, \frac{\vec{J}(\vec{x}')\, e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$$

"$\vec{A}(\vec{x},t)$" $\quad e^{-i\omega t} \quad e^{-i\omega t}$

Example: $\vec{J}(\vec{x}') = I \sin\left(\frac{kd}{2} - k|z'|\right)\delta(x')\delta(y')\,\hat{e}_z$

$|z'| \le d/2$





$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} I \hat{e}_z \int_{-d/2}^{d/2} dz' \frac{\sin\left(\frac{kd}{2} - k|z'|\right)}{|\vec{x}-\vec{x}'|} e^{ik|\vec{x}-\vec{x}'|}$$

no restrictions

$$d \ll r = |\vec{x}|$$

$$|\vec{x}-\vec{x}'| \approx r - \vec{n}\cdot\vec{x}' \,, \quad \vec{n} = \frac{\vec{x}}{|\vec{x}|} \quad \text{(previous lecture)}$$

$$e^{ik|\vec{x}-\vec{x}'|} \approx e^{ikr} e^{-ik\vec{n}\cdot\vec{x}'}$$

$z'\hat{e}_z$

$$\vec{n} = \frac{1}{r}(r\cos\theta\, \hat{e}_z + \cdots)$$

$$\vec{n}\cdot\vec{x}' = z'\cos\theta$$

$$\vec{A}(\vec{x}) = \hat{e}_z \frac{\mu_0 I}{4\pi} \frac{e^{ikr}}{r} \underbrace{\int_{-d/2}^{d/2} dz' \sin\left(\frac{kd}{2} - k|z'|\right) e^{-ikz'\cos\theta}}_{\text{HW (?)}}$$

no dipole approx.

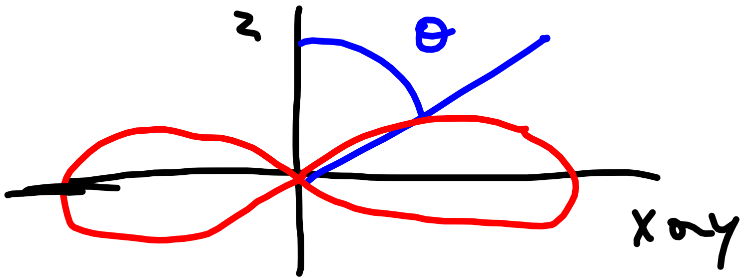
$$\simeq \hat{e}_z \frac{\mu_0 I}{4\pi} \frac{e^{ikr}}{r} \underbrace{\frac{2}{k\sin^2\theta}\left[\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)\right]}_{\text{more complex than just "}\sin\theta\text{"}}$$

$f(\theta, k, d)$

$$\vec{A} \to \vec{H} \to \vec{E} \to \vec{S} \to \frac{dP}{d\Omega}$$

$$\boxed{\frac{dP}{d\Omega} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{I^2}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2}$$

$d \sim \lambda \ll r$



Depends on "kd"

example

$kd = \pi$ (half wave)

$\frac{\omega}{c} d = \frac{2\pi}{\lambda} d = \pi \rightarrow d = \frac{\lambda}{2}$

$\lambda = \frac{2\pi c}{\omega}$

$d \sim \lambda$

example

$kd = 2\pi \rightarrow d = \lambda$ (full wave)

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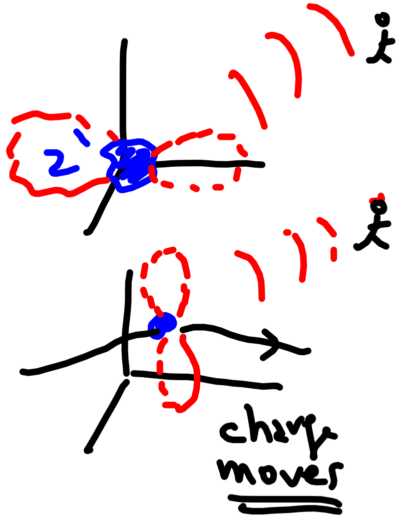
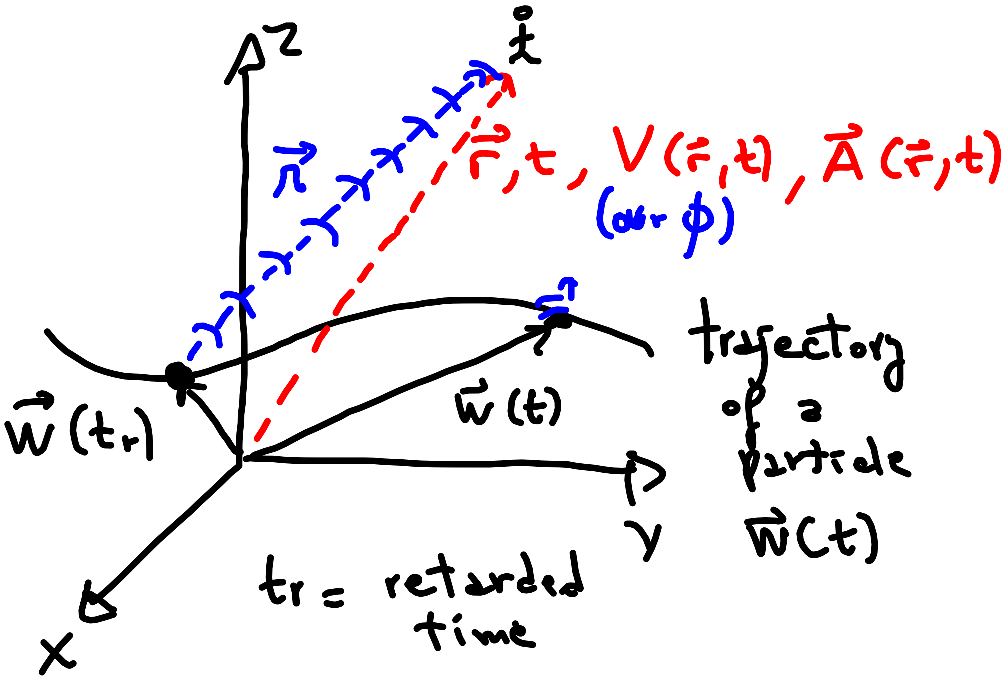
Radiation from a point charge

Griffiths, Sec. 10.3, third edition

Jackson, Ch. 14



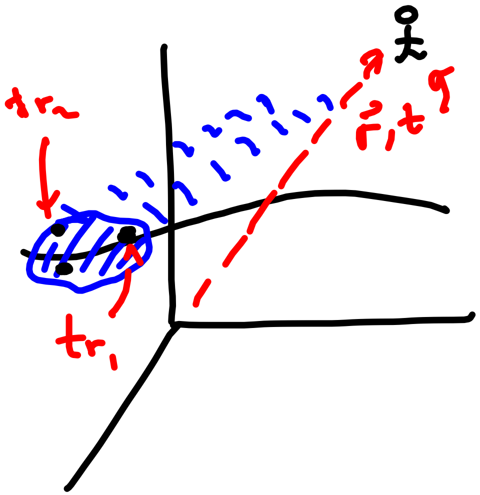


$$|\vec{\mathscr{r}}| = |\vec{r} - \vec{w}(t_r)|$$

$$= c(t - t_r)$$

Example 10.2 Griffiths

$\rho \longrightarrow$ point like charge



$$\phi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{x}',t)_{ret}}{|\vec{x}-\vec{x}'|}\,d\vec{x}'$$

$$V(\vec{r},t) =$$

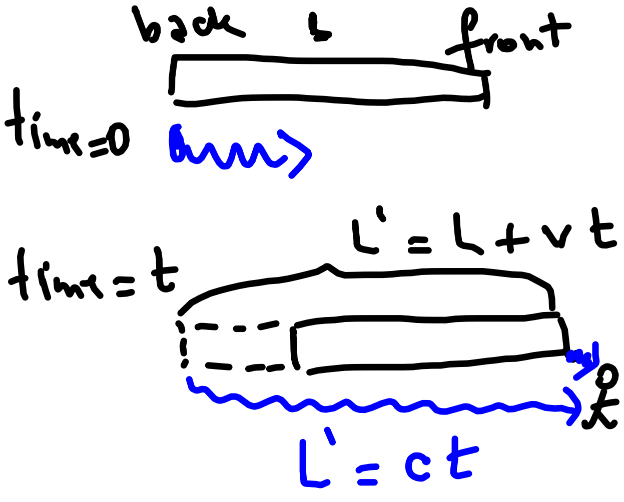
$$= \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}',t_r)\,d^3r'}{|\vec{r}-\vec{w}(t_r)|} \approx \frac{1}{4\pi\epsilon_0|\vec{r}|}\int \rho(\vec{r}',t_r)\,d^3r$$

$\uparrow \vec{r}'$

$$\int \rho(\vec{r}',t_r)\,d^3r' \neq Q$$

changes from point to point

$$\int \rho(\vec{r}',t)\,d^3r' = Q$$

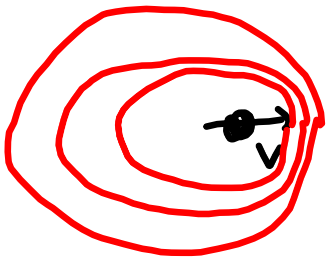


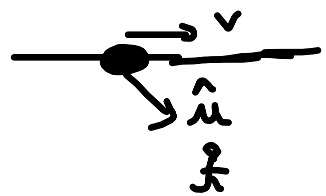
$$t = \frac{L'}{c} = \frac{L'-L}{v}$$

$$\boxed{L' = \frac{L}{1-\frac{v}{c}}}$$

independent of L

- This is ~~not~~ relativity
- Same as Doppler effect



- 

$$\frac{1}{1 - \frac{\hat{u}\cdot\vec{v}}{c}}$$

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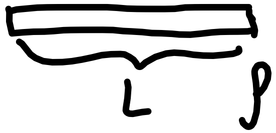
Griffiths



$$\rho = \frac{q}{L} \quad \text{(uniformly distributed)}$$

$$L' = \frac{L}{1-\frac{v}{c}}, \quad \left[q' = \rho L' = q \frac{L'}{L} = q \frac{1}{1-\frac{v}{c}} \right]$$

$$\boxed{V(\vec{r},t) = \frac{q}{4\pi\epsilon_0 |\vec{\mathscr{r}}|} \cdot \frac{1}{\left(1-\frac{\hat{\mathscr{r}}\cdot\vec{v}}{c}\right)}} \xrightarrow[\vec{v}\to 0]{} \frac{q}{4\pi\epsilon_0 |\vec{\mathscr{r}}|}$$