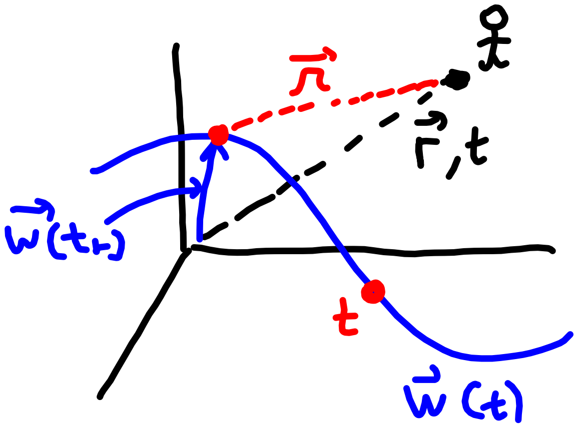
Before

$$\boxed{V(\vec{r},t)} = \underbrace{\frac{q}{4\pi\epsilon_0 \mathscr{r}}}_{\text{as in electrostatics but using the "}t_r\text{" position.}} \cdot \overbrace{\frac{1}{\left(1-\frac{\hat{\mathscr{r}}\cdot\vec{v}}{c}\right)}}^{\text{new factor}} =$$

$\vec{v}(t_r)$

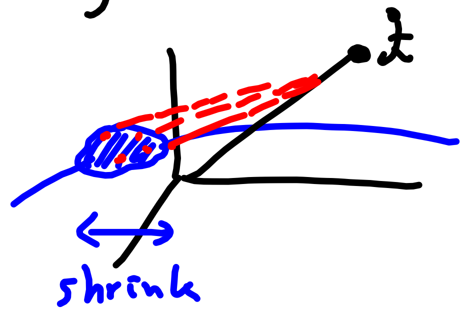


$$= \boxed{\frac{qc}{4\pi\epsilon_0} \cdot \frac{1}{(c\mathscr{r}-\vec{\mathscr{r}}\cdot\vec{v})}}$$

$(\mathscr{r}\hat{\mathscr{r}} = \vec{\mathscr{r}})$

$$\boxed{\vec{A}(\vec{r},t)} = \frac{\mu_0}{4\pi}\int \frac{\rho(\vec{r}\,',t_r)\vec{v}(t_r)d^3r'}{|\vec{\mathscr{r}}|} =$$

$$\vec{J} = \rho\vec{v}$$



$$= \frac{\mu_0}{4\pi}\frac{\vec{v}(t_r)}{|\vec{\mathscr{r}}|}\underbrace{\int\rho(\vec{r}\,',t_r)\,d^3r'}_{q/(1-\frac{\hat{\mathscr{r}}\cdot\vec{v}}{c})}$$

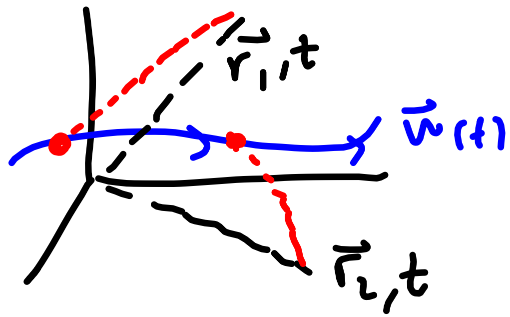
$$\mu_0 = \frac{1}{\epsilon_0 c^2}$$

$$\boxed{= V(\vec{r},t)\frac{\vec{v}(t_r)}{c^2}}$$

$$V,\vec{A} \longrightarrow \vec{E},\vec{B} \text{ via } \nabla V, \frac{\partial\vec{A}}{\partial t}, \ldots \qquad \frac{\partial t_r}{\partial y}$$

Results will depend on $\vec{v}(t_r)$ and $\vec{a}(t_r)$

t_r is a function of $\vec{r}$ and t



Example 10.3 Griffiths

$$\mathcal{R} = |\vec{\mathcal{R}}| \underset{\text{def}}{=} |\vec{r} - \vec{w}(t_r)| = |\vec{r} - \vec{v}t_r|$$

example $\vec{w} = \vec{v}t_r$

$$\mathcal{R}^2 = |\vec{r} - \vec{v}t_r|^2 = (\vec{r} - \vec{v}t_r)\cdot(\vec{r} - \vec{v}t_r)$$

$$c^2(t - t_r)^2 = r^2 - 2\vec{r}\cdot\vec{v}\,t_r + v^2 t_r^2$$

quadratic eq. for t_r

$$t_r = t_r(t, \vec{r}, \vec{v})$$

$$\nabla t_r \neq 0 \qquad \nabla t = 0$$

Special case

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

($\vec{w}$ no longer $\vec{v}t_r$)

$$\nabla V = \nabla\left[\frac{qc}{4\pi\epsilon_0}\frac{1}{(\text{ҝ}c-\vec{\text{ҝ}}\cdot\vec{v})}\right] = \frac{qc}{4\pi\epsilon_0}\left[-\frac{1}{(\text{ҝ}c-\vec{\text{ҝ}}\cdot\vec{v})^2}\right]\nabla(\text{ҝ}c-\vec{\text{ҝ}}\cdot\vec{v})$$

$$\nabla \text{ҝ} = \nabla[c(t-t_r)] = -c\nabla t_r$$

$=0$

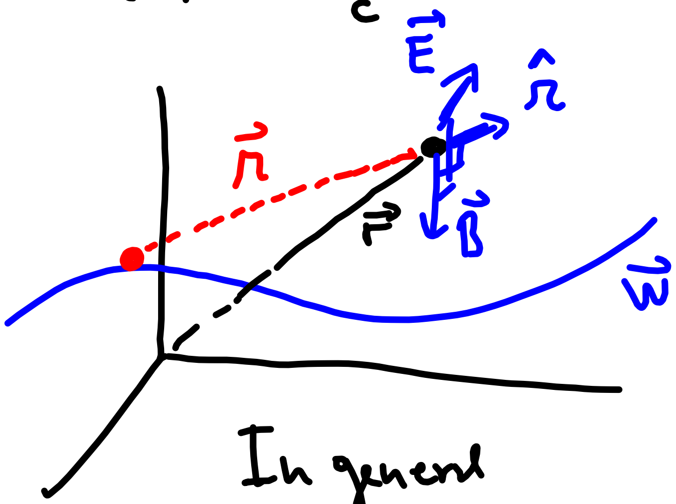
$\neq 0$

$\nabla(\vec{\text{ҝ}}\cdot\vec{v})$ more complex. E.g. $\frac{\partial \vec{v}(t_r)}{\partial x} = \underbrace{\frac{\partial t_r}{\partial x}}_{\nabla t_r|_x}\underbrace{\frac{\partial \vec{v}(t_r)}{\partial t_r}}_{\vec{a}(t_r)}$

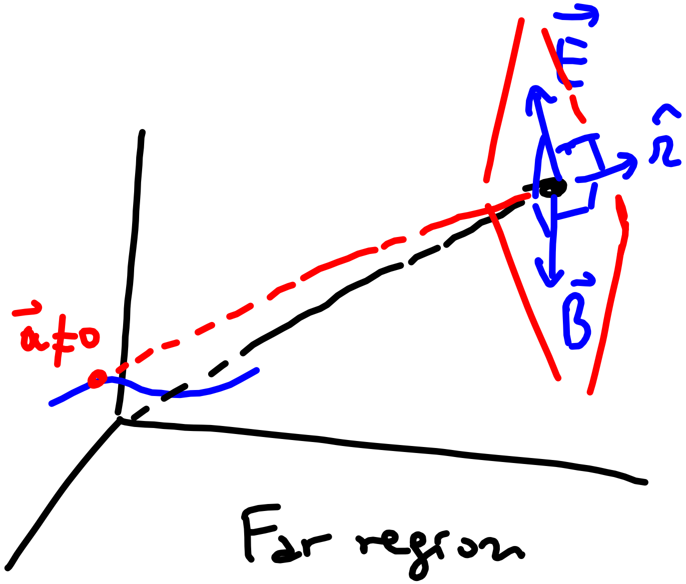
$\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$ $\quad \vec{v}(t_r)$

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0} \underbrace{\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left[\vec{u}(c^2-v^2)\right.}_{\sim \frac{1}{\mathscr{r}^2}} \underbrace{\left.+\vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]}_{\sim\frac{1}{\mathscr{r}}\ \text{(radiation)}} \qquad \left(c\hat{\mathscr{r}} - \vec{v} \underset{\text{def.}}{=} \vec{u}\right)$$

$$\vec{B}(\vec{r},t) = \frac{1}{c}\left[\hat{\mathscr{r}}\times\vec{E}(\vec{r},t)\right]$$



In general

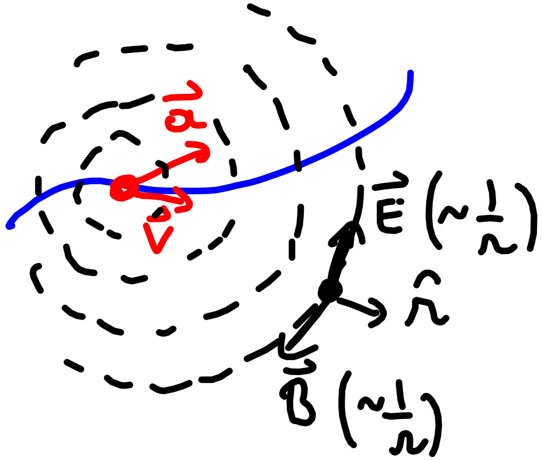


Far region

$\vec{a} = \vec{a}(t_r)$ $\quad\Big\|\quad$ $\vec{\mathscr{r}} \underset{\text{def}}{=} \vec{r} - \vec{w}(t_r)$

$\vec{v} = \vec{v}(t_r)$

Power radiated by a point charge (11.2 Griffiths)



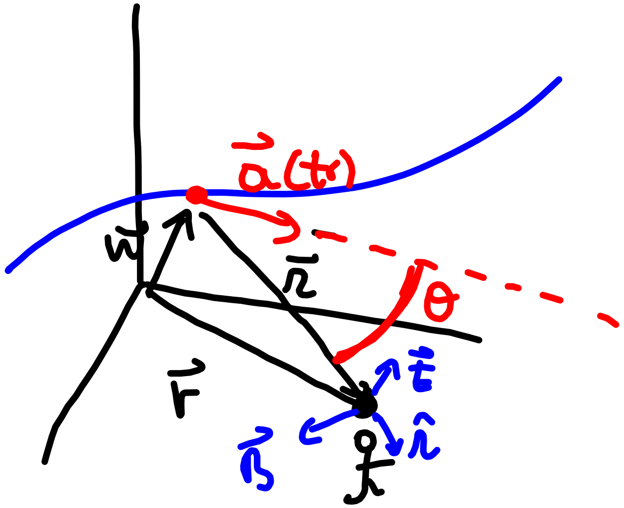
$$\vec{S} = \frac{1}{\mu_0 c} \underbrace{|\vec{E}|^2}_{|\vec{E}|\underbrace{\frac{|\vec{E}|}{c}}_{\sim |\vec{B}|}} \hat{r}$$

Three special cases:

$\vec{v} = 0,\ \vec{a} \neq 0$ Larmor

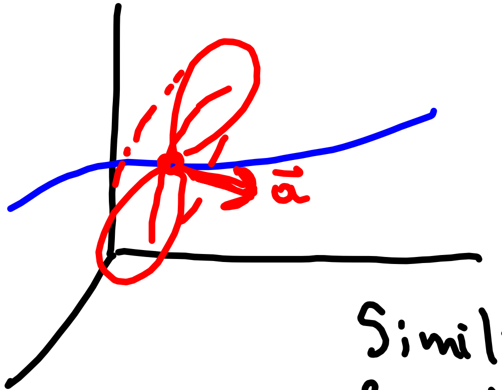
$\vec{v} \neq 0,\ \vec{a} \neq 0,\ \vec{v} \parallel \vec{a}$ Bremsstrahlung

$\vec{v} \neq 0,\ \vec{a} \neq 0,\ \vec{v} \perp \vec{a}$ Synchrotron



$$\vec{S} = \frac{\mu_0 q^2}{16\pi^2 c} \frac{a^2 \sin^2\theta}{r^2} \hat{r}$$

$$\frac{dP}{d\Omega} \sim \vec{S} \cdot \hat{r}\, r^2$$



$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$ (Larmor formula)

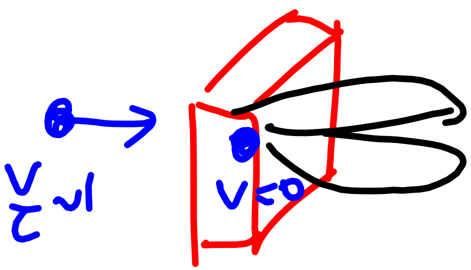
Depends on $|\vec{a}|$, not the sign.

Similar to formula for dipole $\vec{p}$.

$\vec{v} \neq 0, \ \vec{a} \neq 0, \ \vec{v} \parallel \vec{a}$

$$\boxed{\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^2\theta}{\left(1-\frac{v}{c}\cos\theta\right)^5}}$$

new ingredient



very relativistic