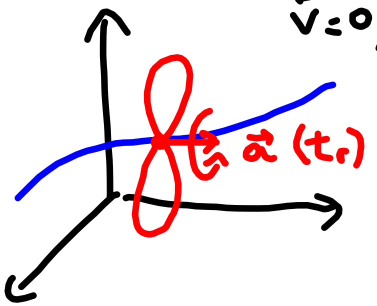
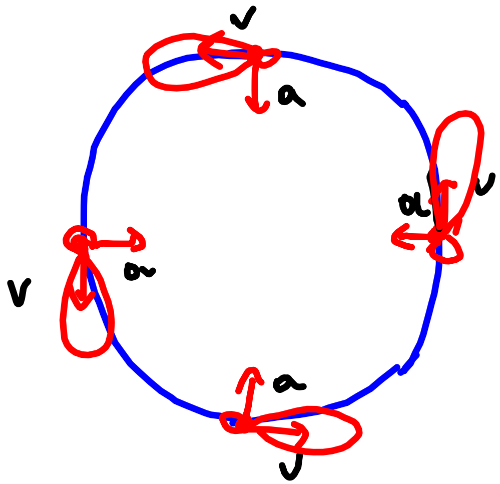
Previous lecture: we found $\vec{E}$ and $\vec{B}$ of a particle with $\vec{v}$ and $\vec{a}$. "Far region" and calculated $dP/d\Omega$ vs. θ.

$\vec{v}=0, \vec{a}\neq 0$



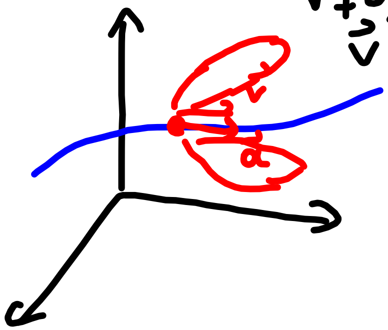
$\vec{v}\neq 0, \vec{a}\neq 0, \vec{v}\perp\vec{a}$



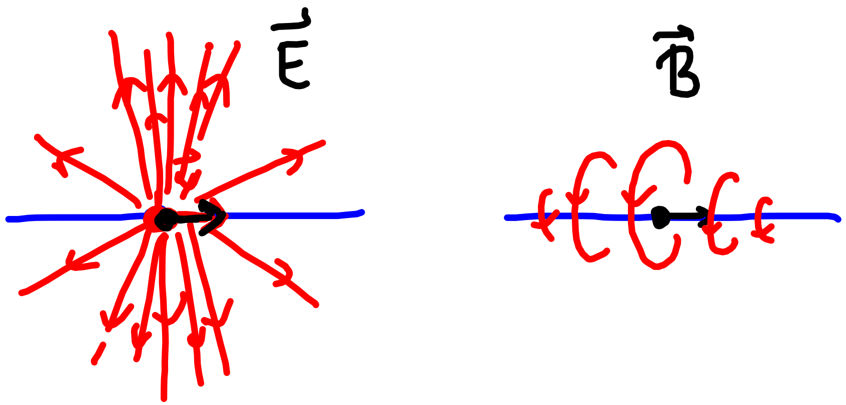
Synchrotron radiation

(train head lights)

$\vec{v}\neq 0, \vec{a}\neq 0$
$\vec{v}\parallel\vec{a}$



$\vec{v} \neq 0, \vec{a} = 0$



$$\varphi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}, t_r)\, d^3r'}{|\vec{r}-\vec{r}'|} = \frac{1}{4\pi\epsilon_0}\int q\,\frac{\delta(\vec{r}'-\vec{w}(t_r))\, d^3r'}{|\vec{r}-\vec{r}'|} =$$

t_{ret} trajectory

$$\neq \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{|\vec{r}-\vec{w}(t_r)|}\;\underbrace{\frac{1}{\left(1-\frac{\hat{n}\cdot\vec{v}}{c}\right)}}_{?}$$

$$\delta(\vec{r}'-\vec{w}(t_r)) \quad t - \frac{1}{c}|\vec{r}-\vec{r}'| \qquad \neq \delta(\vec{r}'-\vec{a})$$

$$\delta(x^2-a^2) = \frac{\delta(x-a)}{2|a|} + \frac{\delta(x+a)}{2|a|}$$ **Example**

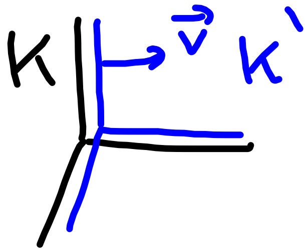
$$\boxed{\delta(f(x)) = \sum_{\substack{i\\ \text{zeros}}} \frac{\delta(x-x_i)}{\left|\frac{\partial f}{\partial x}\right|_{x_i}}}$$

$f(x)=0$

$1-\frac{\hat{n}\cdot\vec{v}}{c}$

Ch. 11 Jackson, Special Theory of Relativity

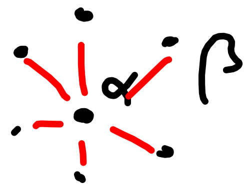
11.1 Galilean transformations



$\vec{x}' = \vec{x} - \vec{v}t$ or $\vec{x} = \vec{x}' + \vec{v}t'$

$t' = t$ $\qquad t = t'$

For particles, the eqs. are invariant



$$m_\alpha \frac{d^2\vec{x}'_\alpha(t')}{dt'^2} = -\nabla'_\alpha \sum_\beta V_{\alpha\beta}\Big(\underbrace{|\vec{x}'_\alpha - \vec{x}'_\beta|}_{\sim \frac{1}{|\vec{x}'_\alpha - \vec{x}'_\beta|}}\Big)$$

$$\Big\downarrow \text{G.T.}$$

$$m_\alpha \frac{d^2\vec{x}_\alpha(t)}{dt^2} = -\nabla_\alpha \sum_\beta V_{\alpha\beta}(|\vec{x}_\alpha - \vec{x}_\beta|)$$

Max. Eqs.

$\rho = 0, \vec{J} = 0$

$$\boxed{\begin{aligned} \nabla^2 \Phi - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} &= 0 \\ \nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} &= 0 \end{aligned}}$$

$$\underbrace{\left(\sum_{i=1}^{3} \frac{\partial^2}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)}_{\text{Is this G. Inv.?}} \psi = 0$$

$\vec{A}, \Phi, \ldots$

$x', t' \longleftrightarrow x, t$

Chain rule; 1D as example

$x = x' + vt'$
$t = t'$

$$\frac{\partial \psi}{\partial t'} = \underbrace{\frac{\partial x}{\partial t'}}_{v} \frac{\partial \psi}{\partial x} + \underbrace{\frac{\partial t}{\partial t'}}_{1} \frac{\partial \psi}{\partial t} = \left(v\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)\psi \; ; \; \boxed{\frac{\partial}{\partial t'} \longrightarrow v\frac{\partial}{\partial x} + \frac{\partial}{\partial t}}$$

$$\frac{\partial \psi}{\partial x'} = \frac{\partial \psi}{\partial x} \; ; \; \boxed{\frac{\partial}{\partial x'} = \frac{\partial}{\partial x}}$$

$$\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} \xrightarrow{GT} \frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\left(v\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)\left(v\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)$$

$$= \frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \underline{\underline{\frac{v^2}{c^2}\frac{\partial^2}{\partial x^2} - \frac{2v}{c^2}\frac{\partial}{\partial x}\frac{\partial}{\partial t}}}$$

Max. Eqs. are not invariant under G. Transf.

What to do next?

- Max. Eqs. are wrong? NO
- Maybe there is an "ether"? NO
- Maybe the G-Transf. is incorrect? YES

Max. Eqs. are invariant under Lorentz transf.

→ "Correct mechanics" must also be Lorentz invariant

11.3

1D as example

$x_0 \overset{\text{def}}{=} ct$

$\beta \overset{\text{def}}{=} \frac{v}{c}$

$\gamma \overset{\text{def}}{=} \frac{1}{\sqrt{1-\beta^2}}$

$$\boxed{\begin{aligned} x_0 &= \gamma x_0' + \gamma\beta x' \\ x &= \gamma x' + \gamma\beta x_0' \end{aligned}}$$

Lorentz transf.

$$\frac{1}{c}\frac{\partial\psi}{\partial t'} = \frac{1}{c}\frac{\partial x}{\partial t'}\frac{\partial\psi}{\partial x} + \frac{c}{c^2}\frac{\partial t}{\partial t'}\frac{\partial\psi}{\partial t}$$

$$\frac{\partial\psi}{\partial x_0'} = \underbrace{\frac{\partial x}{\partial x_0'}}_{\gamma\beta}\frac{\partial\psi}{\partial x} + \underbrace{\frac{\partial x_0}{\partial x_0'}}_{\gamma}\frac{\partial\psi}{\partial x_0}$$

$$\boxed{\frac{\partial}{\partial x'} \xrightarrow{\text{L.T.}} \gamma\frac{\partial}{\partial x} + \gamma\beta\frac{\partial}{\partial x_0}}$$

$$\boxed{\frac{\partial}{\partial x_0'} \xrightarrow{\text{L.T.}} \gamma\beta\frac{\partial}{\partial x} + \gamma\frac{\partial}{\partial x_0}}$$

$$\underbrace{\frac{\partial}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}}_{\text{part of M.Eqs. in primed coord. system}} \xrightarrow{\text{L.T.}} \left(\gamma\frac{\partial}{\partial x} + \gamma\beta\frac{\partial}{\partial x_0}\right)^2 - \left(\gamma\beta\frac{\partial}{\partial x} + \gamma\frac{\partial}{\partial x_0}\right)^2$$

$$= \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{c^2\partial t^2}$$

Max. Eqs. are relativistic