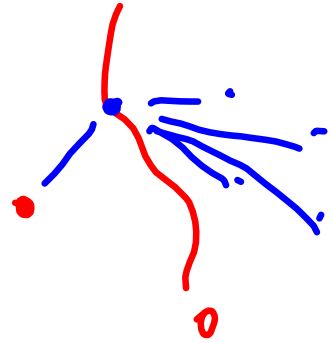
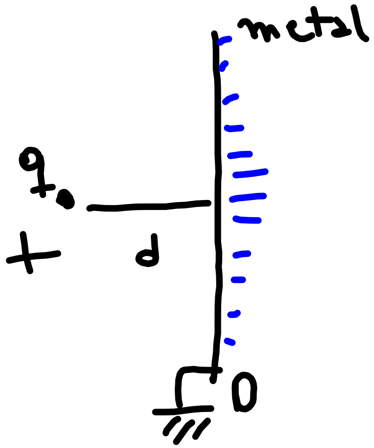
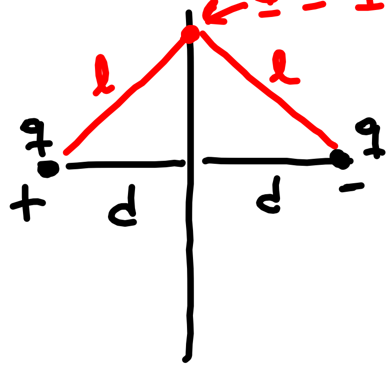


2.1 Method of images

It works in some simple cases.



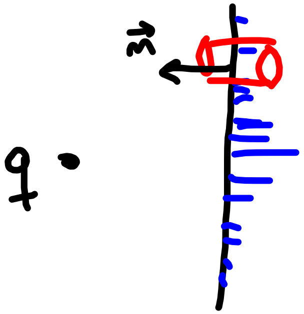


$\Phi = \Phi_q + \Phi_{-q} = 0$

$$\phi = \frac{q}{4\pi\epsilon_0 \, \ell}$$

Both problems are equivalent.

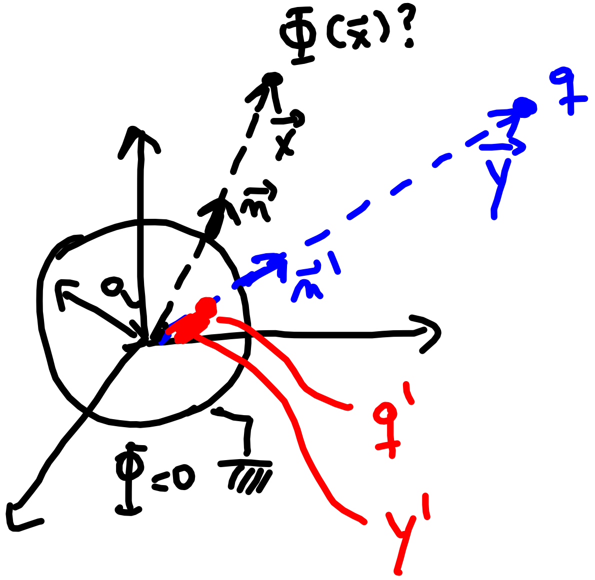
σ : return to original problem



$$\frac{\sigma}{\epsilon_0} = \left(\vec{E}_{out} - \vec{E}_{in}\right)\cdot\vec{n}$$

$$\vec{E}_{out} = -\nabla\left(\Phi_q + \Phi_{-q}\right), \qquad \vec{E}_{in} = 0$$

2.2 Point charge in the presence of a conducting sphere



$$\Phi(\vec{x}) = \underbrace{\frac{q/4\pi\epsilon_0}{|\vec{x}-\vec{y}|}}_{\text{blue charge}} + \underbrace{\frac{q'/4\pi\epsilon_0}{|\vec{x}-\vec{y}\,'|}}_{\text{red charge}} =$$

$|\vec{y}\,'-\vec{x}|$

$$= \frac{q/4\pi\epsilon_0}{x\left|\vec{n}-\frac{y\vec{n}'}{x}\right|} + \frac{q'/4\pi\epsilon_0}{y'\left|\vec{n}'-\frac{x\vec{n}}{y'}\right|}$$

$\vec{x} = x\vec{n}$

$\vec{y} = y\vec{n}'$

$\vec{y}\,' = y'\vec{n}'$

Impose Φ at sphere be zero

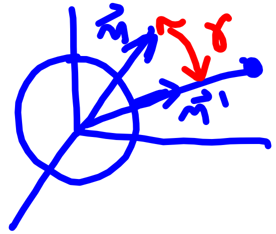
q', y' unknown

$$\Phi(x=a) = 0 = \frac{q/4\pi\epsilon_0}{a\left|\vec{n} - \frac{y}{a}\vec{n}'\right|} + \frac{q'/4\pi\epsilon_0}{y'\left|\vec{n}' - \frac{a}{y'}\vec{n}\right|}$$

$$\sqrt{\left(\vec{n} - \frac{y}{a}\vec{n}'\right)\cdot\left(\vec{n} - \frac{y}{a}\vec{n}'\right)} = \sqrt{1 - \frac{2y}{a}\underbrace{(\vec{n}\cdot\vec{n}')}_{\cos\gamma} + \frac{y^2}{a^2}}$$

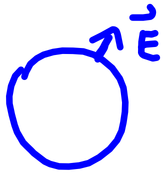
$$\sqrt{1 - \frac{2a}{y'}(\vec{n}\cdot\vec{n}') + \left(\frac{a}{y'}\right)^2}$$

$$\boxed{\frac{a}{y'} = \frac{y}{a} \quad ; \quad \frac{q}{a} = -\frac{q'}{y'}}$$



$$\frac{\sigma}{\epsilon_0} = \left(\vec{E}_{out} - \underbrace{\vec{E}_{in}}_{=0}\right)\cdot\vec{n}$$

$$\vec{E}_{out} = -\nabla\Phi = -\nabla\Phi_{q} - \nabla\Phi_{-q}$$



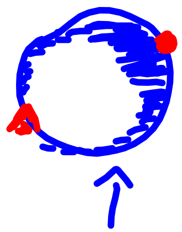
$$\sigma = -\frac{q}{4\pi a y}\,\frac{\overbrace{\left(1-\frac{a^2}{y^2}\right)}^{>0}}{\left(1+\frac{a^2}{y^2}-\frac{2a}{y}\cos\gamma\right)^{3/2}}$$

$\cos\gamma$:

$\vec{n}\cdot\vec{n}' = 1$

$\vec{n}\cdot\vec{n}' = -1$

$+q$



$-$

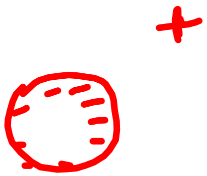
$$\int \sigma\, da = q'$$

Force at q caused by q'.

$$\vec{F} = q \underbrace{\vec{E}_{\text{caused by } q'}}_{-\nabla \Phi_{q'}}$$

attractive

$$|\vec{F}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2}\left(\frac{a}{y}\right)^3 \frac{1}{\left(1-\frac{a^2}{y^2}\right)^2} \xrightarrow[y \gg a]{} \frac{1}{y^3}$$



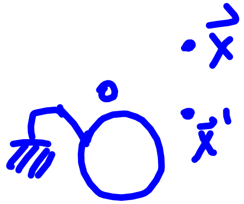
$$\frac{q'}{y^2} \sim \frac{1}{y^3}$$

$q' \sim \frac{1}{y}$

[Work functions of metals]

2.6 Green function of the sphere

Goal: find $G_D(\vec{x},\vec{x}\,') = \dfrac{1}{|\vec{x}-\vec{x}\,'|} + \underbrace{F(\vec{x},\vec{x}\,')}_{\nabla^2 F = 0}$



$$\nabla^2 G_D(\vec{x},\vec{x}\,') = -4\pi\,\delta(\vec{x}-\vec{x}\,')$$

$G_D(\vec{x},\vec{x}\,') = 0$ when $\vec{x}$ is at surface

$$\nabla^2 \Phi_{\vec{x}\,'}(\vec{x}) = -\frac{\rho}{\epsilon_0} = -\frac{q}{\epsilon_0}\,\delta(\vec{x}-\vec{x}\,')$$

$\Phi_{\vec{x}\,'}(\vec{x}) = 0$ at surface

$q = 4\pi\epsilon_0$ (special case)

$G_D = \dfrac{1}{|\vec{x}-\vec{x}\,'|}$ + contribution of the image charge

$$= \frac{1}{\sqrt{x^2+x'^2-2xx'\cos\gamma}} - \frac{1}{\sqrt{\frac{x^2x'^2}{a^2}+a^2-2xx'\cos\gamma}}$$